CONTENTS

Syllabus
TERM I

July

Sets:

Sets and their representations, Empty set, finite & infinite sets, equal sets. Subsets- Subsets of the set of real numbers especially intervals (with notations), Power set, Universal set, Venn diagrams, Union, Intersection, Difference and Complement of a set.

Linear Inequalities:

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Solution of system of linear inequalities in two variables- graphically.

Principle of Mathematical Induction:

Processes of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of Mathematical induction and simple applications.

Relations & Functions:

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the set of real numbers with itself (upto $R \times R \times R$). Definition of relation, pictorial diagrams, domain, co-domain and range of a relation.

August

Relations & Functions(continued):

Function as a special kind of relation from one set to another. Pictorial representation of a function, domain, co-domain & range of a function. Real valued function of the real variable, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum and greatest integer functions with their graphs. Sum, difference, product and quotients of functions.

Trigonometric Functions:

Positive and negative angles. Measuring angles in radians & in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of a unit circle. Truth of the identity $\sin^2 x+\cos^2 x=1$, for all x. Signs of trigonometric functions and sketch of their graphs. Expressing $\sin(x+y)$ and $\cos(x+y)$ in terms of $\sin x, \sin y, \cos x, \cos y$.

Deducing the identities like following:

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}, \tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

$$\cot(x+y)=\frac{\cot x\cot y-1}{\cot x+\cot y}, \cot(x-y)=\frac{\cot x\cot y+1}{\cot x-\cot y}$$

$$\sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x-\sin y=2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x-\cos y=-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

Identities related to $\sin 2x, \cos 2x, \tan 2x, \sin 3x, \cos 3x, \tan 3x$. General solution of trigonometric equations of the type $\sin\theta=\sin\alpha, \cos\theta=\cos\alpha, \tan\theta=\tan\alpha$

September: First Term Examination

TERM II

<u>September - October</u>

Permutations & Combinations:

Fundamental principle of counting. Factorial n. Permutations and combinations, derivation of formulae and their connections, simple applications.

Binomial Theorem:

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, general and middle term in binomial expansion, simple applications.

<u>November</u>

Sequences and Series:

Sequence and Series. Arithmetic progression (A. P.), arithmetic mean (A.M.).Geometric progression (G.P.), general term of a G. P., sum of n terms of a G.P., geometric mean

(G.M.), relation between A.M. and G.M. Sum to infinity of a G.P. Sum to n terms of the special series: $\sum n, \sum n^2, \sum n^3$

Straight Lines:

Brief recall of 2D from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axes, point-slope form, slope-intercept form, two -point form, intercepts form and normal form.

December

Straight Lines (Continued):

General equation of a line. Distance of a point from a line.

Conic Sections:

Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

Introduction to Three -dimensional Geometry:

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula

January

Limits and Derivatives:

Derivative introduced as rate of change both as that of distance function and geometrically, intuitive idea of limit. Definition of derivative, relate it to slope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions. Some important Limits:

$$(i)\lim_{x\to 0}\frac{1}{x},(ii)\lim_{x\to\infty}\frac{1}{x},(iii)\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x},(iv)\lim_{x\to 0}(1+x)^{\frac{1}{x}},(v)\lim_{x\to 0}\frac{\log(1+x)}{x},(vi)\lim_{x\to 0}\frac{e^{x}-1}{x},(vii)\lim_{x\to 0}\frac{a^{x}-1}{x},a>0$$

February

Probability:

Random experiments: outcomes, sample spaces (set representation). Events: occurrence of events, 'not', 'and' & 'or' events, exhaustive events, mutually exclusive events. Axiomatic (set theoretic) probability, connections with the theories of earlier classes. Probability of an event, probability of 'not', 'and' & 'or' events.

Complex Numbers and Quadratic Equations:

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve every quadratic equation. Brief description of algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system. Square root of a complex number.

Mathematics Project Work

Prepare a project on any of the following topics. You may work in groups of not more than four students. Each selected topic must be investigated thoroughly. Relevant information must be collected and understood. Group will be required to make a presentation (not necessarily PowerPoint show, handmade charts or simple displays will also be appreciated).

The assessment will be done at in the second term. The marks will be included in the second term assessment.

The project will be assessed on the basis of mathematical content, depth and clarity of understanding, explanation, neatness and organization, group presentation, coordination, creativity and completion. The final presentation should not exceed ten minutes of time. Please refer to the do's and don'ts of making a digital presentation on page 61.

Topics:

1. **Encryption:** Encryption has been playing a very important role in our lives. From secret messages passed in the ancient wars to the safety of our Gmail account it has been used extensively. Understand the theory behind these systems, how and why of it. Try creating something you can call your own.
2. **Mathematics in Forensics :**Put on your gloves, take out your magnifying glasses and get ready to become a crime scene investigator.They Apply deductive reasoning skills to make sense of the relationships between events, suspects, motives, evidences and ultimately solve this "Who did it"
3. **Queuing Theory:** How is the time and frequency of the metro trains decided? How come the traffic light at one crossing stays red precisely for 3 minutes, while another stays red for merely 40 seconds? Delve into the Queuing theory and find answers to these and other related observations.
4. **Prisoner's Dilemma:** Investigate prisoner' Dilemma and explore its use in marketing strategies.
5. **Mathematical Biology:** What distinguishes us from the others? The answer lies in our genes. Is there anything common between your DNA structure and say, Ramanujan's? Investigate

6. **Konigsberg Bridge Problem:** Understand the problem, its known solution and the theory behind the solution. Can you think of another solution for this problem and its application, for example, in deciding evacuation routes of a hotel or school?
7. **Fractals:** What are Fractals? What are a Sierpinski Triangle and Koch Curve? Discover how to construct the Koch or "Snowflake" Curve and Sierpinski. Learn how to make Fractal Cards – make at least two Fractal Cards.
8. **Let's graph it**!: Explore graphs of various functions, equations and inequalities using Desmos. Think, visualize, create and blend Math with Art.

Sets

1. Let

$U = \{x : x \in N \text{ and } x \leq 8\}$, $A = \{x : x \in N \text{ and } 5 < x^2 < 50\}$,

$B = \{x : x \in N \text{ and } x \text{ is prime no. less than } 10\}$

Verify that $A - B = A \cap B^C$

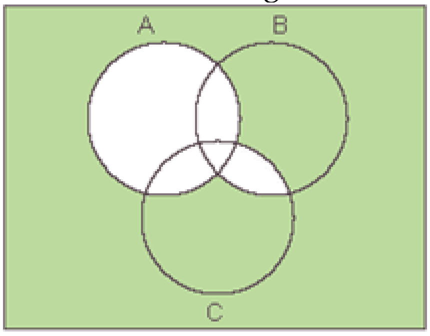
2. Draw Venn diagram of the following sets:

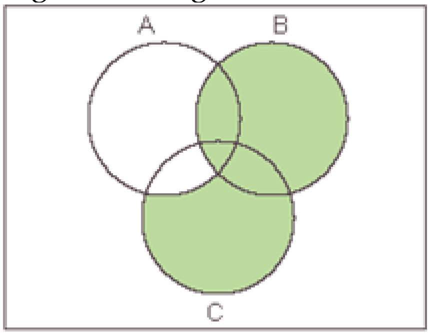
(i) $A' \cap (B \cup C)$ (ii) $A' \cap (C - B)$

3. If U = $\{x: x \in N, and\ x \leq 10\}$, A= $\{x : x\ is\ prime, x \in N\}$ and B=$\{x : x\ is\ a\ factor\ of\ 24, x \in N\}$.

Verify that (i) $A - B = A \cap B'$ (ii) $(A \cup B)' = A' \cap B'$

4. Name the shaded region in the following Venn Diagrams:





5. Out of 100 students, 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many passed
   (i) in English and Mathematics but not Science
   (ii) in Mathematics and Science but not English
   (iii) in Mathematics only
   (iv) in more than one subject

6. A school organised Sports Week during the month of November. Many activities like Athletics, Yoga and Gymnastics were held. Out of 100 students of class XI, 30 participated in Athletics, 35 participated in Yoga and 20 participated in Gymnastics. 12 participated in Athletics and Gymnastics, 8 participated on Yoga and Gymnastics and 10 participated in Yoga and Athletics, while none participated in all three activities. Find
(i) The number of students who did not participate in any of the three activities.
(ii) Number of students who participated in Yoga only
(iii) Number of students who participated in Athletics and Yoga but Gymnastics.

7. In a survey of 100 car owners it was found that 67 owned car A, 46 owned carB and 40 owned car C. 28 owned both A and B, 8 owned both B and C, 26 owned both A and C and 2 owned all the three cars. Find the number of people who owned
   (i) car A but not B and C
   (ii) only two of the cars
   (iii) none of the cars

Assignment No. 1
Sets

Q. No. 1 - 10 are very short answer type questions:

1. Write the set $\left\{\frac{1}{2},\frac{2}{9},\frac{3}{28},\frac{4}{65},\frac{5}{126},\frac{6}{217}\right\}$ in set builder form.
2. Write the set $\left\{\frac{1}{3},\frac{3}{5},\frac{5}{7},\frac{7}{9},\frac{9}{11},\frac{11}{13},\right\}$ in set builder form.
3. Write the set A = $\{x : x \in Z, x^2 < 30\}$ in roster form.
4. Describe the set $A = \{x : x \text{ is a two digit number such that the sum of its digits is } 8\}$ in roster form .
5. Find the pairs of equal sets and equivalent sets from the following : A = $\{1\}$, $B = \{x : x \in R, x > 10 \text{ and } x < 6\}$, $C = \{x : x \in R, x - 1 = 0\}$ $D = \{x : x \in R, x^2 = 36\}$, $E = \{x : x \text{ is an integral root of } x^2 + 8x + 12 = 0\}$
6. Write whether the set is empty, finite or infinite?
   (a) $\{x \in N : x < 200\}$ (b) $\{x \in R, 0 < x < 1\}$ (c) $\{x : 4 < x < 5, x \in N\}$
   (d) $\{x : x \in Z \text{ and } x^2 \text{ is even}\}$ (e) $\{x : x^2 - 5 = 0 \text{ and } x \text{ is rational}\}$
   (f) {Set of circles passing through three non – collinear points}
7. If $A = \{x : x = 5n \ , n \in Z\}$ and $B = \{x : x = 3n \ , n \in Z\}$, find $A \cap B$.
8. If $A = \{1, 2 , \{3,4\}\}$, find $P(A)$.
9. Draw Venn diagrams to represent:
   (a) $A' \cap (C - B)$ (b) $(B - C) \cup (C - B)$ (c) $A - (B \cap C)$ (d) $B' - A'$
10. Let $A = \{a, b, \{c, d\}, e\}$. Which of the following statements are false and why?
    (i) $\{c, d\} \subset A$ (ii) $\{a, b, c\} \subset A$ (iii) $\phi \in A$ (iv) $\{\{c, d\}\} \subset A$
    (v) $\phi \subset A$ (vi) $\{a, b, e\} \in A$ (vii) $b \in A$ (viii) $\{\phi\} \subset A$
11. If $U = \{a, b, c, d, e, f, g\}$, $A = \{b, e, f\}$ and $B = \{a, g\}$,show that $B - A = B \cap A'$.
12. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$, $B = \{3, 5, 7\}$ and $C = \{1, 2, 4, 5, 7\}$ find

(i) $A \cap (B \cup C)'$ (ii) $A' \cup (B \cap C')$ (iii) $(A \cup B)' - (A \cup C)'$ (iv) $(B - A) \cup (A - C)$
(Show steps)

13. In a group of 50 people, 30 like cricket, 25 like football and 32 like hockey. Assume that each one likes at least one of the three games. If 15 people like both cricket and football, 11 like football and hockey and 18 like cricket and hockey, then find how many like (i) all three games (ii) Only football(iii) only hockey

14. In a survey of 100 students, the number of students studying various languages is as follows: only English 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find (i) how many students are studying Hindi? (ii) how many students are studying Hindi and English?

15. In a university, out of 100 students 15 offered Mathematics only; 12 offered statistics only; 8 offered only Physics; 40 offered Physics and Mathematics; 20 offered Physics and Statistics; 10 offered Mathematics and Statistics, 65 offered Physics. Find the number of students who (i) offered Mathematics (ii) offered Statistics (iii) did not offer any of the above three subjects.

16. In a survey 18 people liked Channel A; 23 liked Channel B and 24 liked Channel C. Of these, 13 liked both Channel B and C; 12 Liked Channel A and B; 11 liked Channel C and A and 6 liked all the three Channels. Assuming that everyone liked at least one channel. Find:

    a. how many people were surveyed?

    b. how many people liked Channel C but not B?

    c. how many people liked exactly one of the three channels?

17. Let $X = \{1,2,3,........,10\}$, $A = \{1,2,4,5\}$, $B = \{2,4,6,8,10\}$ and C = $\{2,3,4,5,6,7\}$
    Verify the following results:
    (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (ii) $(A \cap B)' = A' \cup B'$

Learning Objectives
At the end of this unit the student will be able to: • define a set mathematically and represent it in two different ways. • identify an empty set, finite and infinite set. • differentiate between equal sets and equivalent sets. • write the subsets, power sets and universal set of a given set. • find the union and intersection of two or more sets. • appreciate the extensive use and convenience of representing a set and operations on sets pictorially by means of diagrams called Venn diagrams. • differentiate between difference of two sets and complement of a set. • state De Morgan's laws and Complement laws. • apply the concept of set theory and operations on sets to solve real life situations.

Relations and Functions

Q1. Plot the following functions using GeoGebra/ Desmos and explore the following functions:

S. No	FUNCTION	ROUGH SKETCH	DOMAIN	RANGE	CHARACTERISTICS
1	$f(x) = x$				
2	$f(x) = k$ (k: constant)				
3	$f(x) = \frac{1}{x}$				
4	$f(x) = x^2$				
5	$f(x) = -x^2$				
6	$f(x) = (x-1)^2 + 3$				
7	$f(x) = \sqrt{x}$				

8	$f(x) = \sqrt{4 - x^2}$				
9	$f(x) = -\sqrt{4 - x^2}$				
10	$f(x) = \sqrt{x^2 + 9}$				
11	$f(x) = \lvert x \rvert$				
12	$f(x) = -\lvert x \rvert$				
13	$f(x) = \frac{\lvert x \rvert}{x}$				
14	$f(x) = \frac{\lvert x + 2 \rvert}{x + 2}$				

15	$f(x) = \frac{x+1}{2x+3}$				
16	$f(x) = \frac{1}{1+x^2}$				
17	$f(x) = x^3$				
18	$f(x) = \frac{x}{1+x^2}$				
19	$f(x) = \frac{x^2}{1+x^2}$				
20	$f(x) = \frac{1}{1+2x}$				

Q2. Find the domain and range of all the functions in question 1 algebraically.

Q3. Draw the rough sketch of the following functions:

i) $f(x) = \begin{cases} x+1, & -1 \leq x \leq 1 \\ 3-x, & 1 \leq x \leq 3 \\ 6-2x, & 3 \leq x \leq 4 \end{cases}$

ii) $f(x) = \begin{cases} x-1, & x < 2 \\ 2x+3, & x \geq 2 \end{cases}$

iii) $f(x) = |1+x| + |1-x|, x \in [-2,2]$

iv) $f(x) = |x+3|$

v) $f(x) = \begin{cases} 3+2x, & x \geq 0 \\ x+3, & x < 0 \end{cases}$

vi) $f(x) = \begin{cases} x-1, & x \geq 0 \\ 3x, & x < 0 \end{cases}$

vii) $f(x) = \begin{cases} x+1, & x > 2 \\ 4, & x = 2 \\ 2x+1, & x < 2 \end{cases}$

viii) $f(x) = \begin{cases} 2x+1, & x \geq 3 \\ x^2, & x < 3 \end{cases}$

ix) $f(x) = \begin{cases} 1-x, & x < -3 \\ x^2, & -3 \leq x \leq 4 \\ 4x, & x \geq 4 \end{cases}$

x) $f(x) = \begin{cases} 5-x, & x < -1 \\ x^2, & -1 \leq x < 2 \\ 2x, & x \geq 2 \end{cases}$

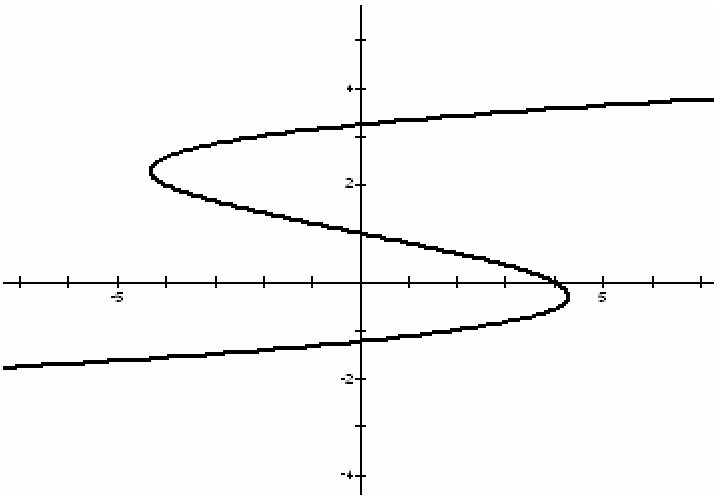
xi) $f(x) = 2x - |x|$

xii) $f(x) = |x-2| - 1$

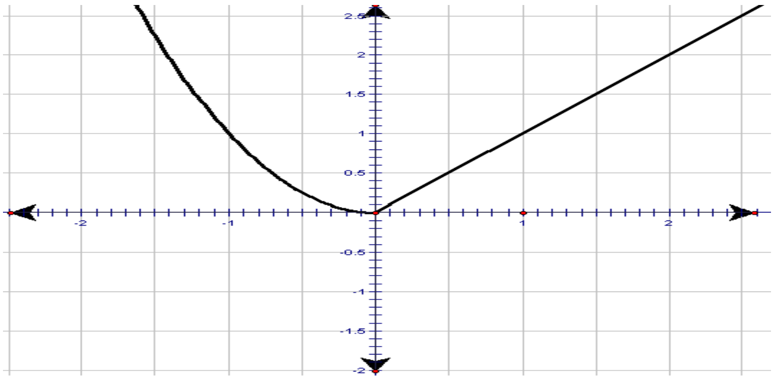
Assignment No. 2
Relations and Functions

Q. No. 1 - 13 are very short answer type questions:

1. If A= {2,3}, B= {4,5} and C= {5,6}, find $A \times (B \cap C)$.
2. Is the following graph, the graph of a function of x? Justify.



3. Is the following graph, the graph of a function of x ? Justify.



4. Let $A=\{2,3\}, B=\{4,5\}$ .Find the total number of relations from A into B.
5. Determine the domain and range of the relation R defined by $R=\{(a,b): b=|a-1|, a \in Z \text{ and } |a| \le 3\}$
6. Draw the graph of the function $y=|x-3|$.
7. Determine the domain and range of the relation R defined by $R=\{(x+1, x-1): x \in \{1,2,3,4,5,6\}\}$
8. If $x, y \in \{1,2,3,4\}$, then is $f=\{(x,y): x+y>4\}$ a function? Justify.
9. Find the range of the function $f(x)=\dfrac{3-x}{x-3}$.

10. Let $f : R \to R$, be defined as $f(x) = x^2 + 1$, then find the pre-image of 17.

11. If $f(x) = x^2 - 3x + 1$ and $f(2\alpha) = 2f(\alpha)$, then find the value of α.

12. Let $A = \{2,3,4,5,6\}$.Let R be the relation on A defined by the rule "$(x, y) \in R$ iff x divides y". Find R as a subset of $A \times A$.

13. Let $f : R \to R$ and $g : N \to N$ be two functions defined as $f(x) = x^2$ and $g(x) = x^2$. Are they equal functions?

14. Find the domain and range of the following functions:

    (i) $1-|x-3|$ (ii) $\frac{3}{2-x^2}$ (iii) $\sqrt{9-x^2}$ (iv) $\frac{1}{\sqrt{x-3}}$

15. If f is the identity function and g is the modulus function, find $f+g, f-g, f.g, \frac{f}{g}$

16. Let f, g be two real functions defined by $f(x) = 4(x-1)^2$ and $g(x) = 4x^2$. Then describe each of the following functions (NOTE: Do mention domains of new functions described).

    (i) $g - f$ (ii) $\frac{f}{g}$ (iii) $2f - \sqrt{g}$ (iv) $\frac{4}{g}$

17. Draw the graph of the following function $f(x) = \begin{cases} -x-1 \text{ if } x < -2 \\ x+1 \text{ if } -2 \le x \le 1 \\ 3 \text{ if } x > 1 \end{cases}$

Learning Objectives

At the end of this unit the student will be able to:

- define an ordered pair and write the Cartesian product of two sets and extend it to n sets.
- define a relation between two sets, find the domain, range and codomain of the relation and represent the relation pictorially with the help of arrow diagrams.
- define a function from one set to another set.
- identify some standard functions, draw the graphs of the functions.
- find the domain, range and co domain of the functions by drawing the graphs of the functions.
- find the domain, range and co domain of the functions algebraically.
- find the sum, difference, product and quotient of the two functions.

Trigonometric Functions

1. If $\sin\theta = \frac{3}{5}, \tan\varphi = \frac{1}{2}, \frac{\pi}{2} < \theta < \pi < \varphi < \frac{3\pi}{2}$ then find the value of $8\tan\theta - \sqrt{5}\sec\varphi$.
$\left(A = \frac{-7}{2}\right)$
2. If $\sin A = \frac{3}{5}, 0 < A < \frac{\pi}{2}, \cos B = \frac{-12}{13}, \pi < B < \frac{3\pi}{2}$, find the value of $\sin(A-B), \cos(A+B), \tan(A-B)$
3. If A lies in the fourth quadrant and $\cos A = \frac{5}{13}$, find the value of $\frac{13\sin A + 5\sec A}{5\tan A + 6\cos ecA}$
4. If $\cos\theta = \frac{-1}{2}$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of $4\tan^2\theta - 3\cos^2\theta$
5. If $\sin A = \frac{3}{5}, 0 < A < \frac{\pi}{2}$ and $\cos B = \frac{-12}{13}, \pi < B < \frac{3\pi}{2}$ then find the following:
(i) $\sin(A-B)$ (ii) $\cos(A+B)$ (iii) $\tan(A-B)$ $\left(A = \frac{-16}{65}, \frac{-33}{65}, \frac{16}{63}\right)$
6. Find the value of $\tan(A+B)$, given that $\cot A = \frac{1}{2}, \sec B = \frac{-5}{3}, \pi < A < \frac{3\pi}{2}, \frac{\pi}{2} < B < \pi$

7. Prove the following:
   a) $\cos 570°\sin 510° + \sin(-330°)\cos(-390°) = 0$
   b) $\frac{\cos(2\pi+\theta)\cos ec(2\pi+\theta)\tan\left(\frac{\pi}{2}+\theta\right)}{\sec\left(\frac{\pi}{2}+\theta\right)\cos(\theta)\cot(\pi+\theta)} = 1$
   c) $\frac{\cos(90°+\theta)\sec(-\theta)\tan(180°-\theta)}{\sec(360°-\theta)\sin(180°+\theta)\cot(90°-\theta)} = -1$
   d) $\sin^2\frac{\pi}{4} + \sin^2\frac{3\pi}{4} + \sin^2\frac{5\pi}{4} + \sin^2\frac{7\pi}{4} = 2$
   e) $\sin 600°\tan(-690°) + \sec 840°\cot(-945°) = \frac{3}{2}$
   f) $\cos 306° + \cos 234° + \cos 162° + \cos 18° = 0$
   g) $\sin^2 54° - \sin^2 72° = \sin^2 18° - \sin^2 36°$

8. In any quadrilateral ABCD, prove that: $\sin(A+B) + \sin(C+D) = 0$
9. Prove that:
   a) $\tan 315°\cot(-405°) + \cot 495°\tan(-585°) = 2$
   b) $\cos 510°\cos 330° + \sin 390°\cos 120° = -1$
   c) $\sin\frac{8\pi}{3}\cos\frac{23\pi}{6} + \cos\frac{13\pi}{3}\sin\frac{35\pi}{6} = \frac{1}{2}$
   d) $\cos 570°\sin 510° + \sin(-330°)\cos(-390°) = 0$

e) $3\sin\frac{\pi}{6}\sec\frac{\pi}{3}-4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}=1$

f) $\frac{\cos(2\pi+\theta)\operatorname{cosec}(4\pi+\theta)\tan\left(\frac{\pi}{2}+\theta\right)}{\sec\left(\frac{\pi}{2}+\theta\right)\cos(-\theta)\cot(\pi+\theta)}=1$

10. In any cyclic quadrilateral ABCD, prove:
    a) $\tan A+\tan B+\tan C+\tan D=0$
    b) $\cos(180°-A)+\cos(180°+B)+\cos(180°+C)-\sin(90°+D)=0$
11. Find x from the following equation: $\operatorname{cosec}(90°+\theta)+x\cos\theta\cot(90°+\theta)=\sin(90°+\theta)$
12. If A,B,C,D are angles of a cyclic quadrilateral, prove that:
    $\cos A+\cos B+\cos C+\cos D=0$
13. Find x from the following equation:
    a) $\operatorname{cosec}(270°+A)=\cos(180°+A)+x\sin(90°+A)\cot(270°+A)$
    b) $x\cot(90°+A)+\tan(90°+A)\sin A+\operatorname{cosec}(90°+A)=0$
14. Prove that:
    a) $\tan 8\theta-\tan 6\theta-\tan 2\theta=\tan 8\theta\tan 6\theta\tan 2\theta$
    b) $\tan 70°=\tan 20°+2\tan 50°$
    c) $\frac{\cos 11°+\sin 11°}{\cos 11°-\sin 11°}=\tan 56°$
    d) $\frac{\cos 15°-\sin 15°}{\cos 15°+\sin 15°}=\frac{1}{\sqrt{3}}$
15. If $A+B=\frac{\pi}{4}$, prove that $(1+\tan A)(1+\tan B)=2$
16. Evaluate : a) $\cos(-1125°)$ b) $\tan\left(\frac{11\pi}{6}\right)$ c) $\operatorname{cosec}(-1200°)$
17. Prove that :
    a) $\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}=\tan A$
    b) $\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}=0$
    c) $\tan 13\theta-\tan 9\theta-\tan 4\theta=\tan 13\theta\tan 9\theta\tan 4\theta$
    d) $\tan 15°+\tan 30°+\tan 15°\tan 30°=1$
    e) $\tan 36°+\tan 9°+\tan 36°\tan 9°=1$
    f) $\frac{\cos 8°-\sin 8°}{\cos 8°+\sin 8°}=\tan 37°$
    g) $\frac{\cos 9°+\sin 9°}{\cos 9°-\sin 9°}=\tan 54°$
    h) $\tan 50°=\tan 40°+2\tan 10°$
    i) $\tan\left(\frac{\pi}{4}+A\right)\tan\left(\frac{\pi}{4}-A\right)=1$

j) $(1+\tan A)(1+\tan B) = 2\tan A, \textit{where } A - B = \frac{\pi}{4}$

k) $\tan 75° - \tan 30° - \tan 75° \tan 30° = 1$

18. If $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$, the prove that : $\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = 1$

19. Prove that:

a) $\frac{1+\sin 2\theta + \cos 2\theta}{1+\sin 2\theta - \cos 2\theta} = \cot\theta$

b) $\frac{\cos 2\theta}{1+\sin 2\theta} = \tan\left(\frac{\pi}{4} - \theta\right)$

c) $\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \cot\frac{\theta}{2}$

d) If $\tan\frac{x}{2} = \frac{n}{m}$, find the value of $m\cos x + n\sin x$

20. Show that: $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

21. Prove that :

a) $\frac{\sin 2\theta}{1-\cos 2\theta} = \cot\theta$

b) $\frac{1+\sin\theta - \cos\theta}{1+\sin\theta + \cos\theta} = \tan\frac{\theta}{2}$

c) $\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} = \tan\theta$

d) $\sqrt{2+\sqrt{2+2\cos 4\theta}} = 2\cos\theta$

e) $\frac{\sin 2\theta + \sin\theta}{1+\cos\theta + \cos 2\theta} = \tan\theta$

f) $\cos 4\theta = 1 - 8\sin^2\theta + 8\sin^4\theta$

22. If $\sin x = \frac{\sqrt{5}}{3}$ and $\frac{\pi}{2} < x < \pi$, find the value of $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$.

23. Prove that : $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$

24. Prove that: $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right) = \frac{1}{8}$

25. Prove that : $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = \frac{3}{2}$

26. Prove that: $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

27. Prove that :

a) $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

b) $\dfrac{\sin x-\sin y}{\cos x+\cos y}=\tan\left(\dfrac{x-y}{2}\right)$

c) $\dfrac{\cos x-\cos 2x+\cos 3x}{\sin x-\sin 2x+\sin 3x}=\cot 2x$

d) $\dfrac{(\cos x-\cos 3x)(\sin 8x+\sin 2x)}{(\sin 5x-\sin x)(\cos 4x-\cos 6x)}=1$

e) $\dfrac{\sin A+2\sin 3A+\sin 5A}{\sin 3A+2\sin 5A+\sin 7A}=\dfrac{\sin 3A}{\sin 5A}$

f) $\dfrac{\sin 3A+\sin 5A+\sin 7A+\sin 9A}{\cos 3A+\cos 5A+\cos 7A+\cos 9A}=\tan 6A$

g) $\dfrac{\sin(x+y)-2\sin x+\sin(x-y)}{\cos(x+y)-2\cos x+\cos(x-y)}=\tan x$

h) $\cos 40^\circ+\cos 50^\circ+\cos 70^\circ+\cos 80^\circ=\cos 10^\circ+\cos 20^\circ$

i) $\cos\theta-\cos 3\theta+\cos 5\theta-\cos 7\theta=4\sin\theta\sin 4\theta\cos 2\theta$

j) $\sin\theta+\sin\left(\dfrac{2\pi}{3}+\theta\right)+\sin\left(\dfrac{4\pi}{3}+\theta\right)=0$

k) *If* $\dfrac{\sin x}{a}=\dfrac{\cos x}{b}$, prove that $a\sin 2x+b\cos 2x=b$

28. Prove that: $\dfrac{\cos 8A\cos 5A-\cos 12A\cos 9A}{\sin 8A\cos 5A+\cos 12A\sin 9A}=\tan 4A$

29. Prove that: $\sin 10^\circ\sin 30^\circ\sin 50^\circ\sin 70^\circ=\dfrac{1}{16}$

30. Prove that: $\cos 10^\circ\cos 30^\circ\cos 50^\circ\cos 70^\circ=\dfrac{3}{16}$

31. Prove the following :

a) $\dfrac{\sin 3A\cos 2A-\sin 6A\cos A}{\sin A\sin 2A-\cos 2A\cos 3A}=\dfrac{\sin 3A}{\cos A}$

b) $\dfrac{\sin 3A\cos 4A-\sin A\cos 2A}{\sin 4A\sin A+\cos 6A\cos A}=\tan 2A$

c) $\dfrac{\sin 2A\sin 3A-\sin A\sin 4A+\sin 5A\sin 2A}{\cos 4A\cos 5A-\cos 3A\cos 6A+\cos 5A\cos 2A}=\tan 4A\tan 3A$

d) $\cos 20^\circ\cos 40^\circ\cos 60^\circ\cos 80^\circ=\dfrac{1}{16}$

e) $\sin 10^\circ\sin 50^\circ\sin 60^\circ\sin 70^\circ=\dfrac{\sqrt{3}}{16}$

f) $\tan 20^\circ\tan 30^\circ\tan 40^\circ\tan 80^\circ=1$

32. Find the general solutions of the following equations :

(i) $\sqrt{3}\cos\theta+\sin\theta=\sqrt{2}$

(ii) $\cos x+\sin x=\sqrt{2}$

(iii) $\cot\theta + \cos ec\theta = \sqrt{3}$

(iv) $\sin 2x = \cos 3x$

(v) $4\cos x - 3\sec x = \tan x$

(vi) $\tan x + \tan 2x + \tan x \tan 2x = 1$

(vii) $\cos x + \cos 3x - 2\cos 2x = 0$

33. Find the general solutions of the following equations :

a) $\cos 3\theta = -\frac{1}{2}$

b) $\tan\left(\frac{2}{3}\theta\right) = \sqrt{3}$

c) $\tan^2\theta + \left(1-\sqrt{3}\right)\tan\theta - \sqrt{3} = 0$

d) $\tan x + \tan 2x + \tan 3x = \tan x \tan 2x \tan 3x$

e) $2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

f) $\sin x + \sin 2x + \sin 3x = 0$

g) $\cos x + \cos 3x - \cos 2x = 0$

h) $\sqrt{3}\cos x + \sin x = 1$

i) $\cos ecx = 1 + \cot x$

Assignment No. 3
Trigonometric Functions

Q. No. 1 - 8 are very short answer type questions:

1. Find the degree measure of an angle through which a pendulum swings if its length is 50 cm and the tip describes an arc of length 16 cm $\left(\text{Take } \pi = \frac{22}{7}\right)$.

2. Find the value of $\operatorname{cosec}\left(\frac{-19\pi}{3}\right)$.

3. Prove that $\cos\theta - \sin\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)$

4. Find the general solution to the equation $\cos\theta = -\frac{1}{2}$.

5. Find the principal solution to the equation $\cot\theta = -\sqrt{3}$.

6. Prove that $\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$.

7. Find the value of $\cot\left(\frac{37\pi}{12}\right)$.

8. Prove the following:

$(i)\ \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$

$(ii)\ \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right) = 0$

$(iii)\ \frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = \tan 4A$

(iv) $\tan 11\theta - \tan 7\theta - \tan 4\theta = \tan 11\theta \tan 7\theta \tan 4\theta$

(v) $\frac{\cos 2A \cos 3A - \cos 2A \cos 7A + \cos A \cos 10A}{\sin 4A \sin 3A - \sin 2A \sin 5A + \sin 4A \sin 7A} = \cot 6A \cot 5A$

9. If $\cos A = \frac{-24}{25}$ and $\sin B = \frac{-4}{5}$, where $\pi < A < \frac{3\pi}{2}$ and $\frac{3\pi}{2} < B < 2\pi$, find the following:

   (i) $\sin(A+B)$ (ii) $\tan(A-B)$

10. Prove that: $\cos 10^\circ \cos 50^\circ \cos 60^\circ \cos 70^\circ = \frac{\sqrt{3}}{16}$

11. Find the general solution for each of the following equations :

$(i)\sin 2x+\sin 4x+\sin 6x=0$

$(ii)\sqrt{2}\sec\theta+\tan\theta=1$

$(iii)\cot^2\theta+\dfrac{3}{\sin\theta}+3=0$

$(iv)\sin 3\theta+\cos 2\theta=0$

12. Prove the following:

$(i)\cos^2 A+\cos^2\left(A+120^\circ\right)+\cos^2\left(A-120^\circ\right)=\dfrac{3}{2}$

$(ii)\dfrac{\tan\left(\dfrac{\pi}{4}+\theta\right)+\tan\left(\dfrac{\pi}{4}-\theta\right)}{\tan\left(\dfrac{\pi}{4}+\theta\right)-\tan\left(\dfrac{\pi}{4}-\theta\right)}=\cos ec2\theta$

$(iii)\dfrac{\sec 8\theta-1}{\sec 4\theta-1}=\dfrac{\tan 8\theta}{\tan 2\theta}$

$(iv)\cos^2\dfrac{\pi}{8}+\cos^2\dfrac{3\pi}{8}+\cos^2\dfrac{5\pi}{8}+\cos^2\dfrac{7\pi}{8}=2$

13. If $\tan x=\dfrac{4}{3}$ and $\pi<x<\dfrac{3\pi}{2}$, find the values of $\sin\dfrac{x}{2},\cos\dfrac{x}{2},\tan\dfrac{x}{2}$

14. If $\tan(A+B)=m$ and $\tan(A-B)=n$, show that $\tan 2B=\dfrac{m-n}{1+mn}$.

Learning Objectives
At the end of this unit the student will be able to: • define negative angle, radian measure and convert angles from radians to degree and vice versa. • extend the definition of trigonometric ratios to any angle in terms of radian measure and study them as trigonometric functions with the help of a unit circle. • determine the sign of trigonometric functions. • find the domain and range of trigonometric functions. • draw the graphs of trigonometric functions. • find the values of trigonometric functions in different quadrants. • find trigonometric functions of sum and difference of two angles and derive cos (x +y). • deduce identities of the form sin (x + y), sin 2x, sin 3x and so on using cos (x +y). • solve problems based on the above deduced identities. • define principal solution and general solution of a trigonometric equation. • solve a trigonometric equation. • deduce half angle formula and solve problems based on them.

Linear Inequalities

1) Solve: $-12x > 30$, when (i) x is a natural number (ii) x is an integer
2) Given set A= $\{x \in Z : 0 \le x \le 10\}$. Solve the inequality $-2x+6 \le 5x-4$ in set A.

3) Find the greatest value of x which satisfies the inequality:

$$x+\frac{x}{2}+\frac{x}{3}<11, \text{ where } x \in Z$$

4) Solve: a) $|x-1|>5$ b) $\frac{1}{x-2}<0,\ x \in R$

5) Solve:

(i) $\frac{x-5}{x+2}<0$ (ii) $\frac{x+3}{x+5}>5$ (iii) $|3x-2|\le\frac{1}{2}$ (iv) $\left|\frac{4x-5}{3}\right|\le\frac{5}{3}$

$(v)\ \frac{5x+8}{4-x}<2$ $(vi)\ \frac{2x+4}{x-1}\ge 5$ $(vii)\ \frac{4x+3}{2x-5}<6$ $(viii)\ \left|\frac{2x-5}{3}\right|\le 1$

6) Solve the following system of inequalities and represent the solution on number line

(i) $2(2x+3)-10<6(x-2),\ \frac{2x-3}{4}+6\ge 2+\frac{4x}{3}$

(ii) $\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4},\ \frac{7x-1}{3}-\frac{7x+2}{6}>x$

Assignment No. 4

Linear Inequalities

Each part of Q. No 1 and Q. No. 2- 4 are very short answer type questions:

1. If a, b, c are real numbers such that $a \le b, c > 0$, then:

   (i) $ac \le bc$ (ii) ac <bc (iii)ac >bc (iv) $ac \ge bc$. (Choose the correct option)

2. Solve for x: $3x + 9 \ge -x + 19$
3. Solve: $3x - 4 < 7$,when $x \in Z$
4. Solve the following system of inequalities: $2x - 3 < 7, 2x > -4$
5. How many litres of a 30% acid solution must be added to 500 litres of a 12% solution so that acid content in the resulting mixture will be more than 14% but less than 20%.
6. Solve: $6x + 2 < 4x + 7$, when (i) x is a natural number (ii) x is an integer (iii) x is a real number and represent solution for each part on the number line.
7. Find all pairs of consecutive even positive integers, both of which are larger than 7, such that their sum is less than 30.
8. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 7.2 and 7.8. If the first two pH readings are 7.48 and 7.85, find the range of pH value for the third reading that will result in the acidity level being normal.
9. Solve the following linear inequalities and show the graph of solution in each case on the number line :($x \in R$)

   (i) $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2$ (ii) $|3x - 7| > 4$ (iii) $\frac{5x+8}{4-x} < 2$

   (iv) $\left|\frac{3x-4}{2}\right| \le \frac{5}{12}$ (v) $3x - 2 > x + \frac{4-x}{3} > 3$

10. Solve the following system of inequalities:

    (i) $5x - 7 < 3(x+3)$, $1-\frac{3x}{2} \ge x - 4$

    (ii) $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$, $\frac{2x-1}{12} - \frac{x-11}{3} < \frac{3x+1}{4}$

11. Solve the following system of inequalities graphically:

    (i) $3x + 4y \ge 12$, $4x + 7y \le 28$, $y \ge 1$, $x \ge 0$, $y \ge 0$

    (ii) $x + 2y \le 10$, $x + y \ge 1$, $x - y \le 0$, $x \ge 0$, $y \ge 0$

    (iii) $x + 2y \le 40$, $3x + y \ge 30$, $4x + 3y > 60$, $x \ge 0$, $y \ge 0$

Learning Objectives
At the end of this unit the student will be able to: • solve a linear inequality in one variable algebraically and represent the solution on a number line. • find the graphical solution of a linear inequality in two variables. • find solution of a system of linear inequalities in two variables graphically. • solve simple word problems related to real life situations using the concept of inequalities.

Assignment No. 5
Complex Numbers and Quadratic Equations

Q. No. 1 - 5 are very short answer type questions:

1. Find the value of x and y $(x, y \in R)$ if : $2y+(3x-y)i = 5-2i$
2. Express $3i^3+6i^{16}-7i^{29}+4i^{27}$ in the form $x+iy$ where $x, y \in R$.
3. Evaluate : $\left(i^{41}+\dfrac{1}{i^{257}}\right)^9$
4. If $Z_1 = 1-i, Z_2 = -2+4i$, find $\text{Im}\left(\dfrac{Z_1 Z_2}{\bar{Z}_1}\right)$.
5. Find the conjugate of the complex number: $\dfrac{1}{2-3i}$
6. Write the following complex numbers in the polar form:
   (i) $-2-2i$ (ii) $\dfrac{1}{1+i}$
7. Find the complex conjugate of $\dfrac{(8-3i)(6-i)}{2-2i}$.
8. Find the multiplicative inverse of $\left(\dfrac{3+4i}{4-5i}\right)$
9. Find the modulus and argument of $\dfrac{1+2i}{1-3i}$
10. If $(a+ib)^2 = (x+iy)$, prove that $(a^2+b^2)^2 = (x^2+y^2)$
11. Find x and y if $\dfrac{(1+i)x-2i}{3+i}+\dfrac{(2-3i)y+i}{3-i} = i$
12. For what values of x and y are the numbers $-3+ix^2y$ and x^2+y+4i complex conjugates? (x, y are real numbers.)
13. Solve the following quadratic equations:
(i) $6x^2-17ix-12=0$
(ii) $3x^2+7ix+6=0$
(iii) $x^2-(7-i)x+18-i=0$
(iv) $x^2-(3\sqrt{2}-2i)x-6\sqrt{2}i=0$
(v) $2x^2-(3+7ix)x+9i-3=0$

14. Find the square root of: (i) $-8-6i$, (ii) $-5+12i$, (iii) $-i$

Learning Objectives
At the end of this chapter the student will be able to: • define complex number. • understand Algebra of Complex Number. • find modulus and conjugate of a complex number. • understand Argand Plane and Polar Representation.

Permutations and Combinations
Permutations

1. How many even numbers of 3 digits can be formed with the digits 1,2,3,4,6 if no digit is repeated?
2. How many numbers of six digits can be formed from the digits 0,1,3,5,7 and 9, which are divisible by 10 and no digit is repeated?
3. Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.
4. How many numbers greater than 1000 and less than 4000 can be formed with the digits 0,1,2,3,4 if (a) repetition of digits is allowed (b) repetition of digits is not allowed?
5. How many odd numbers greater than 80000 can be formed using the digits 2,3,4,5 and 8 if each digit is used only once in a number?
6. Three dice are rolled. Find the number of possible outcomes in which at least one die shows a 5.
7. Evaluate: (a) $12!-10!$ (b) $\frac{9!}{5!\times 4!}$
8. Which of the following is true? (a) $(2+3)!=2!+3!$ (b) $(2\times 3)!=2!\times 3!$
9. Find x, if : $\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$
10. If ${}^{11}P_r={}^{12}P_{r-1}$, find r.
11. Find r, if $5\ {}^{4}P_r=6\ {}^{5}P_{r-1}$
12. If ${}^{2n-1}P_n : {}^{2n+1}P_{n-1}=22:7$, find n.
13. How many words can be made from the letters of the word MONDAY assuming that no letter is repeated if (a) 4 letters are used at a time (b) All letters are used at a time? (c) All letters are used but the first letter is a vowel?
14. The letters of the word TRIANGLE are arranged in such a way that vowel and consonants remain together. How many different arrangements will be obtained?
15. Four different mathematics books, six different physics books and two different chemistry books are to be arranged on a shelf. How many different arrangements are possible if (a) the book in a particular subject must all stand together (b) only the mathematics books must stand together?
16. In how many ways can 5 children be arranged in a row such that two boys Akash and Samir are (a) always together (b) never together?
17. How many different 8 letter words can be formed out of the letters of the word DAUGHTER so that (a) The word starts with D and ends with R (b) Position of H remains unchanged (c) Relative position of vowels and consonants remain unaltered (d) No two vowels are together (e) All vowels never occur together?
18. How many words can be formed with the letters of the word EQUATION? In how many of them (a) vowels occur together (b) the vowels never occur together (c) the vowels and the consonants are together?
19. In how many ways can 5 boys and 3 girls be seated in a row so that no two girls are together?

20. In how many ways can 6 girls and 4 boys be arranged in a queue if no two boys are to stand together?
21. How many words can be formed out of the letters of the word ORIENTAL so that A and E occupies odd places?
22. There are 8 candidates appearing in a test. 3 of them have to appear in Mathematics and the remaining in 5 different subjects. How many seating arrangements are possible if they are to sit in a row and no two Mathematics students sit together?
23. The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary. Find the rank of the word RANDOM. What is the 50^{th} word?
24. If the different permutations of all the letter of the word EXAMINATION are listed as in a dictionary, how many words are there in this list before the first word starting with E?
25. In how many ways can 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?
26. A coin is tossed 6 times. In how many ways can we obtain 4 heads and 2 tails?
27. How many number of arrangements are possible with the letters of the word (a) PINEAPPLE (b) INDEPENDENCE?
28. In how many different ways the letters of the word ALGEBRA can be arranged in a row if(a) the two A's are together (b) the two A's are not together ?
29. In how many ways can the letters of the word PERMUTATIONS be arranged such that (a) there is no restriction (b) words start with P and end with S (c) T's are together (d) vowels are together (e) there are always 4 letters between P and S?
30. Find the number of arrangements that can be made of the letters of the word MATHEMATICS. In how many of them (a) M is at both the extremes (b) vowels always occur together (c) vowels are never together (d) no two vowels are together?
31. How many permutations of the letters of the word MADHUBANI do not begin with M but end with I?
32. How many numbers greater than 10,00,000 can be formed from the digits 1,2,0,2,4,2,4?
33. In how many of the distinct permutations of the letters in MISSISSIPPI do four I's not come together?
34. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's occur together?
35. Five boys and five girls form a line with the boys and girls sitting alternately. In how many ways can they be seated?

Combinations

36. If ${}^{n}C_{18} = {}^{n}C_{12}$, then find the value of ${}^{32}C_{n}$
37. If ${}^{18}C_{r} = {}^{18}C_{r+2}$, find ${}^{r}C_{5}$
38. If ${}^{n}P_{r} = 720$ and ${}^{n}C_{r} = 120$, find r.
39. From a group of 15 cricket players, a team of 11 players is to be chosen. In how many ways can this be done?
40. How many different selection of 4 books can be made from 10 different books, if (a) there is no restriction (b) two particular books are always selected (c) two particular books are never selected?
41. A student has to answer 10 questions, choosing at least 4 from each of part A and part B. If there are 6 questions in part A and 7 in part B, in how many ways can the student choose 10 questions?
42. Find the number of diagonals of a (a) hexagon (b) a polygon of 16 sides
43. There are 15 points in a plane, no three of which are in the same straight line except four which are collinear. Find the number of a) straight lines b) triangles formed by joining them.
44. In how many ways can a committee of 5 persons be formed out of 6 men and 4 women when (a) at least two women has to be selected (b) at most 2 women are selected (c) exactly two women are selected?
45. From 4 officers and 8 jawans, in how many ways can 6 be chosen if (a) it has to include exactly one officer (b) it has to include at least one officer?
46. How many different words, each containing 2 vowels and 3 consonants can be formed with 5 vowels and 17 consonants?
47. A tea party is arranged for 16 people along two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated?
48. What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these :

    (a) four cards are of the same suit (b) four cards belong to four different suits (c) are face cards (d) two are red cards and two are black cards (e) cards are of the same colour.

Assignment No. 6
Permutations and Combinations

Q. No. 1- 5 are very short answer type questions:

1. If ${}^nC_{10} = {}^nC_{12}$, find ${}^{23}C_n$.
2. If ${}^{16}C_r = {}^{16}C_{r+2}$, find ${}^rC_4$.
3. If ${}^{11}P_r = {}^{12}P_{r-1}$, find r.
4. In an examination, a student is to answer 4 questions out of 5. Question 1 and 2 are, however compulsory. Determine the number of ways in which the student can make the choice.
5. How many three-digit numbers are there, with distinct digits, with each digit odd?
6. In how many ways can the 6 boys and 5 girls be arranged for a group photograph if the girls are to sit on chairs in a row and the boys are to stand in a row behind them?
7. How many numbers between 400 and 1000 can be formed with the digits 0, 2, 3, 5, 6, 7 if no digit is repeated in the same number?
8. Find the number of ways in which 5 boys and 5 girls be seated in a row so that :
   (i) no two girls may sit together.
   (ii) all the girls sit together and all the boys sit together.
9. How many permutations can be formed by the letters of the VOWELS, when
   (i) there is no restriction on letters?
   (ii) each word starts with O and ends with L?
   (iii) all vowels come together?
   (iv) all vowels never come together?
10. Out of 5 boys and 3 girls, a committee of 5 is to be formed. In how many ways can it be done if the committee contains
    (i) exactly two girls?
    (ii) at least two girls?
    (iii) at most two girls?
11. How many words can be formed with the letters of the word UNIVERSITY, the vowels remaining together?
12. How many numbers greater than a million can be formed with the digits 2, 3, 0, 7, 7, 3, 7?
13. In an examination a candidate has to pass in each of the four subjects. In how many ways can he fail?

14. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in a dictionary. Prove that the word SACHIN appears at serial number 601.
15. How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that the odd digits always occupy the odd places?

Learning Objectives
At the end of this unit the student will be able to: • define the fundamental principle of counting. • define factorial n. • define permutation mathematically and simple application based on permutation. • define combination and understand the difference between permutation and combination. • simple applications on combination.

Assignment No. 7
Binomial Theorem

Each part of Q. No. 1 & 2 and Q. No. 3- 8 are very short answer type questions:

1. Write down the general term in the expansion of

   $(i)\left(1-x^3\right)^{11}, (ii)\left(x^2-\frac{1}{x}\right)^{12}$.

2. Find the middle term(s) in the expansion of $\left(\frac{4x}{5}-\frac{5}{2x}\right)^9$.

3. Find the 5th term from the end in the expansion of $\left(\frac{x}{2}+\frac{2}{x}\right)^{10}$.

4. Find the 13th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$.

5. In the expansion of $(1+x)^{2n}$ the coefficients of $(p+1)th$ and $(p+3)th$ terms are equal, prove that $p=n-1$.

6. For what value of m, the coefficients of the $(2+m)th$ and $(4m+5)th$ terms in the expansion of $(1+x)^{10}$ are equal.

7. Find the coefficient of:

   (i) x^{-3} in the expansion of $\left(2x^2+\frac{1}{x}\right)^{12}$

   (ii) x^2 in the expansion of $\left(3x-\frac{1}{x}\right)^6$.

8. Find the term independent of x in the expansion of:

   (i) $\left(2x-\frac{1}{x}\right)^{10}$ (ii) $\left(3x-\frac{1}{x}\right)^6$

10. Show that the ratio of the coefficients of x^{10} in the expansion of $\left(1-x^2\right)^{10}$ and the absolute term in the expansion $\left(x-\frac{2}{x}\right)^{10}$ is 1:32.

11. If the coefficients of the three successive terms in the expansion of $(1+x)^n$ are 462, 330 and 165, find n.

12. Using Binomial Theorem, show that $3^{2n+2}-8n-9$ is divisible by 64 for all Natural numbers n.

13. If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^{2n}$ are in A.P., show that $2n^2 - 9n + 7 = 0$.

14. Prove that there is no term involving x^{-1} in the expansion of $\left(\frac{x^2}{2}+\frac{1}{x}\right)^{12}$.

15. Find the coefficient of x^5 in the expansion of $(1+x)^3(1-x)^6$.

Learning Objectives
At the end of the chapter the student will be able to: • expand a binomial expression raised to the power of a natural number. • write the general term, term independent of x and specific terms of an expansion. • establish relation between coefficients between terms and find missing values. • prove the binomial theorem using the Principle of Mathematical Induction.

Sequences and Series

1. If a,b,c are in A.P. show that $\frac{a(b+c)}{bc},\frac{b(c+a)}{ca},\frac{c(a+b)}{ab}$ are in A.P.

2. If $a\left(\frac{1}{b}+\frac{1}{c}\right),b\left(\frac{1}{c}+\frac{1}{a}\right),c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P. show that a,b,c are in A.P.

3. Show that if the positive numbers a,b,c are in A.P., so are the numbers $\frac{1}{\sqrt{b}+\sqrt{c}},\frac{1}{\sqrt{c}+\sqrt{a}},\frac{1}{\sqrt{a}+\sqrt{b}}$.

4. If a,b,c,d are in G.P, prove that $(a-b)^2,(b-c)^2,(c-d)^2$ are in G.P.

5. If a,b,c,d are in G.P, prove that a^2+b^2,b^2+c^2,c^2+d^2 are in G.P.

6. Find the sum to n terms of the series:

   i. 5 + 11 + 19 + 29 + 41……….

   ii. 3+ 7 +13 +21 +31 +…………….

   iii. 1+ 3 + 7 + 15 + 31 + …………..

   iv. 3 + 15 + 35 + 63 + …………….

   v. 5 + 7 + 13 + 31 + 85+ …………

Assignment No. 8
Sequences and Series

Q. No 1- 7 are very short answer type questions:

1. Write the value of the 10th term of the sequence: $1(1)+2(1+2)+3(1+2+3)+.....$

2. The 4th term a G.P. is x, the 10th term is y and the 16th term is z. Write the relation between x, y and z.

3. Insert three geometric means between 2 and 32.

4. Which term of the series: $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....$ is $\frac{1}{512}$?

5. The third term of a G. P. is 4, find the product of first 5 terms.

6. Evaluate: $\sum_{1}^{20}\left(2^{n}+5^{n-1}\right)$.

7. Insert two A.M.'s between 2 and 3.

8. (i) If a^2, b^2, c^2 are in A.P., then show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are also in A.P.

   (ii) $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P., then prove that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P.

9. The ratio of the sum of n terms two A.P.'s is $(3n+4):(5n+6)$, , find the ratio of their 5th terms.

10. There are n A.M.'s between 3 and 17. The ratio of the last mean to the first mean is 3:1. Find the value of n.

11. The product of three numbers in a G.P. is 216 and the sum of the product of the numbers taken in pairs is 156. Find the numbers.

12. Three numbers are in A.P. and their sum is 15. If 1, 3, 9 be added to them respectively, they form a G.P. Find the numbers.

13. Find the sum to n terms of the series:

$(i) 2+5+10+17+26+.....$

$(ii) 2+5+11+23+47+.....$

14. Find the sum to n terms of the series:

$$(i) \frac{1^3}{1}+\frac{1^3+2^3}{2}+\frac{1^3+2^3+3^3}{3}+.....$$

$(ii) 1.2^2+3.3^2+5.4^2+7.5^2+.....$

15. The first term of a G.P. is 2 and sum to infinity is 6. Find the common ratio.

16. The sum of an infinite G.P. is 15 and the sum of their squares is 45. Find the G.P.

17. The first term of G.P. exceeds the 2^{nd} term by 2 and the sum to infinity is 50. Find the G.P.

Learning Objectives
At the end of the chapter the student will be able to: • define a sequence, series and progression, finite and infinite series. • find specific terms and nth term of a given sequence. • find the nth term and specific terms of an AP. • find the sum to n terms of an AP. • insert arithmetic means between two numbers. • show a given set of three numbers to be in AP when three other numbers are in AP. • define GP and find the terms of a GP. • find sum to n terms of a GP. • insert GM between numbers and find the sum to infinite numbers which are in GP. • show a given set of three numbers to be in GP when three other numbers are in GP. • solve problems based on AP and GP and relation between AM and GM. • find the sum of first ‘n natural numbers, sum of squares of first ‘n natural numbers and sum of cubes of first ‘n’ natural numbers. • find the sum of n terms of series involving sum of first ‘n natural numbers, sum of squares of first ‘n natural numbers and sum of cubes of first ‘n’ natural numbers.

Assignment No. 9
Straight Lines

Q. No. 1 - 7 are very short answer type questions:

1. Reduce $x+\sqrt{3}y+12=0$ to the normal form and hence find the distance of the line from the origin.
2. Find the distance between the parallel lines $2x-3y+9=0$ and $4x-6y+1=0$.
3. A line passes through the points $(4,-6)$ and $(-2,-5)$. Does it make an acute angle with the positive direction of x-axis?
4. Determine x so that the inclination of the line containing the points $(x,-3)$ and $(2,5)$ is $135°$.
5. Find the equation of the line with slope -1 and whose perpendicular distance from the origin is equal to 5.
6. Find the equation of the line passing through the point $(-4,3)$ and parallel to y-axis.
7. At what point must the origin be shifted, if the coordinates of a point (-4,2) becomes (3,-2)?
8. A line passes through (7, 9) and the portion of it intercepted between the axes is divided by this point in the ratio 3 : 1. Find the equation of the line.
9. Find the equation of the line which passes through the point (-3, -2) and cuts off intercepts on x and y axes which are in the ratio 4:3
10. If $A(1,4), B(2,-3), C(-1,-2)$ are the vertices of ΔABC, find the equation of: (i) the median through A. (ii) the altitude through A. (iii) right bisector of BC.
11. Find the angles of a triangle whose sides are x + 2y – 8 = 0; 3x + y – 1 = 0 and x – 3y + 7 = 0.
12. Find the equation of the line such that the area of the triangle formed by the line and the coordinate axes in the first quadrant is 30 and length of the hypotenuse is 13.
13. Prove that line 5x – 2y – 1 = 0 is mid parallel to the lines 5x – 2y – 9 = 0 and

    5x – 2y + 7 – 0.

14. Find the circumcenter of the triangle whose vertices are $(3,0)$, $(-1,-6)$ and $(4,-1)$.

15. Find the equations of straight lines which are perpendicular to the line $3x + 4y - 7 = 0$ and are at a distance of 3 units from (2, 3).
16. Find the equations of the lines which pass through $(4,5)$ and make an angle of $45°$ with the line $2x + y + 1 = 0$.
17. Find the coordinates of the foot of the perpendicular from the point $(2,3)$ on the line $x = 3y + 4$.
18. Find the image of the point (4, -13) in the line mirror $5x + y + 6 = 0$.
19. If the image of the point $(2,1)$ in a line is (4, 3), then find the equation of the line.
20. Find the equation of the line passing through the intersection of the lines $3x + y - 9 = 0$ and $4x + 3y - 7 = 0$ and perpendicular to the line $5x - 4y + 1 = 0$.
21. If the origin is shifted to the point $(3,-1)$, find the new equation of the straight line $2x - 3y + 5 = 0$.
22. The line $2x - 3y = 4$ is the perpendicular bisector of the line segment AB. If coordinates of A are (-3, 1), find coordinates of B.

Learning Objectives
At the end of this chapter the student will be able to: • define slope of a line. • find slope of a line given its inclination; given two points on the line. • determine condition for two lines to be perpendicular/parallel of in terms of slope. • find condition of collinearity of three points. • find angle between two intersecting lines. • to form equation of a line given: (i) a point and slope (ii) two points (iii) slope and y-intercept (iv)x and y intercepts (v) distance of origin from the line and angle made by the perpendicular with origin. • find Distance of a point from a line. • find Distance between two parallel lines. • solve related problems from exercises and miscellaneous exercise.

Assignment No.10
Conic Sections

Q. No. 1- 3 are very short answer type questions:

1. Find the equation of the ellipse whose end points of major axis are $(0,\pm\sqrt{5})$ and of minor axis are $(\pm 1,0)$.
2. Find the equation of the circle with centre $(-3,2)$ and radius 4.
3. Find the radius of the circle $x^2+y^2+8x+10y-8=0$.
4. Find the equation of a circle concentric with the circle $2x^2+2y^2-6x+8y+1=0$ and of double its area.
5. Show that the points $A(1,0), B(2,-7), C(8,1)$ and $D(9,-6)$ all lie on the same circle. Find the equation of this circle, its centre and radius.
6. Find the equation of the circle circumscribing the triangle formed by the lines $x+y=6, 2x+y=4, x+2y=5$
7. Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum:

   (i) $y^2=-6x$ (ii) $x^2=8y$

8. Find the equation of the parabola that satisfies the given conditions:

   (i) Focus is $(0,-4)$; directrix is $y=4$

   (ii) Vertex is $(0,0)$ and it passes through the point $(3,2)$ and is symmetric with respect to x-axis.

9. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse:

   (i) $16x^2+25y^2=400$ (ii) $16x^2+9y^2=144$

10. Find the equation for the ellipse with centre at (0,0) that satisfies the given conditions:

    (i) $e=\frac{1}{2}$ and foci are $(\pm 2,0)$.

    (ii) $e=\frac{2}{3}$, major axis along x-axis and length of latus rectum = 5.

11. Find the coordinates of the foci, the vertices , the eccentricity and the length of the latus rectum of the hyperbolas:

(i) $25x^2 - 36y^2 = 225$ (ii) $16y^2 - 4x^2 = 1$

12. Find the equation for the hyperbola with center at (0,0) that satisfies the given conditions:

(i) distance between whose foci is 32 and whose eccentricity is $2\sqrt{2}$.
(ii) $e = 3$, transverse axis along x-axis and length of latus rectum is 4 units.

Learning Objectives
At the end of the chapter the student will be able to: • define a circle, parabola, ellipse and hyperbola using the definition of locus of a moving point. • visualise each of the above as a conic section. • derive the general second-degree equation of a circle and find equation of circles with given conditions. • derive the equation of a parabola symmetric to x or y axes and find out the focus and equation of directrix. • derive the equation of an ellipse and find out lengths of major axis, minor axis, latus rectum, the foci, the vertices and value of eccentricity. • derive the equation of a hyperbola and find out lengths of conjugate axis, transverse axis, latus rectum, the foci, the vertices and value of eccentricity.

Assignment No. 11
Three Dimensional Geometry

Q. No. 1- 5 are very short answer type questions:

1. The points $(4,7,8),(2,3,4)$ and $(-1,-2,1)$ are the three vertices of a parallelogram. The fourth vertex of the parallelogram is : (a) $(1,2,-5)$ (b) $(1,2,5)$ (c) $(1,-2,5)$ (d) $(-1,2,5)$
2. Find the points on the y-axis which are at a distance of 3 units from the point $(2,3,-1)$.
3. Find the coordinates of the point which divides the join of $(1,-2,3)$ and $(3,4,-5)$ in the ratio 2 : 3 externally.
4. Show that the points $(1,3,2),(3,0,8)$ and $(9,-2,5)$ are the vertices of an isosceles right angled triangle.
5. Show that the points $A(-2,3,5)$, $B(1,2,3)$ and $C(7,0,-1)$ are collinear. Find the ratio in which C divides AB.
6. Find the ratio in which the YZ-plane divides the segment joining the points $(1,2,4)$ and $(3,8,6)$. Also find the coordinates of the point of intersection.
7. Two vertices of a triangle are $(4,-6,3)$ and $(2,-2,1)$ and its centroid is $\left(\frac{8}{3},-1,2\right)$. Find the third vertex.
8. Show that the points $A(2,3,-2)$, $B(6,9,-4)$, $C(7,0,-1)$ and $D(3,-6,1)$ taken in order are the vertices of a parallelogram.
9. A point C with z-coordinate 8 lies on the line segment joining the points $A(2,-3,4)$ and $B(8,0,10)$. Find its coordinates.
10. Find the point on the z-axis which is equidistant from the points $(3,2,1)$ and $(5,2,5)$.

Learning Objectives
At the end of this unit the student will be able to: • understand Coordinate axes and Coordinate Planes in Three-Dimensional Space. • appreciate coordinates of a point in space. • find distance between two points in space. • apply Section Formula in space.

Limits

1. $\lim_{x\to -1}\frac{x^3-3x+1}{x-1}$

2. $\lim_{x\to \frac{1}{2}}\frac{8x^3-1}{16x^4-1}$

3. $\lim_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{x}{x^3-1}\right)$

4. $\lim_{x\to 3}\left(x^2-9\right)\left(\frac{1}{3+x}+\frac{1}{x-3}\right)$

5. $\lim_{x\to 2}\frac{\sqrt{x^2-1}+\sqrt{x-1}}{\sqrt{x^2+1}}$

6. $\lim_{x\to 4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

7. $\lim_{x\to 1}\frac{(2x-3)\left(\sqrt{x}-1\right)}{2x^2+x-3}$

8. $\lim_{x\to 0}\frac{\sqrt{1+3x}-\sqrt{1-3x}}{x}$

9. $\lim_{x\to a}\left(\frac{x^m-a^m}{x^n-a^n}\right)$

10. $\lim_{x\to a}\left(\frac{x^5-a^5}{x-a}\right)=405$, find all possible values of a.

11. $\lim_{x\to 2}\left(\frac{x^5-32}{x^3-8}\right)$

12. $\lim_{x\to a}\frac{(x+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{x-a}$

13. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

14. $\lim_{x\to 1}\frac{1-x^{\frac{-1}{3}}}{1-x^{\frac{-2}{3}}}$

15. $\lim_{x\to \frac{1}{4}}\frac{4x-1}{2\sqrt{x}-1}$

16. $\lim_{x\to 0}\frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}$

17. $\lim_{x\to 9}\frac{x^{\frac{3}{2}}-27}{x-9}$

18. $\lim_{x\to 4}\frac{x-4}{3-\sqrt{13-x}}$

19. $\lim_{x\to \frac{-1}{2}}\frac{8x^3+1}{2x+1}$

20. $\lim_{x\to 5}\frac{(x-3)^5-32}{x-5}$

21. $\lim_{x\to 1}\frac{x^4-3x^3+2}{x^3-5x^2+3x+1}$

22. $\lim_{x\to 1}\frac{x^3+3x^2-6x+2}{x^3+3x^2-3x-1}$

23. $\lim_{x\to 1}\frac{x^2-\sqrt{x}}{\sqrt{x}-1}$

24. $\lim_{x\to 3}\frac{x^2-9}{\sqrt{3x-2}-\sqrt{x+4}}$

25. $\lim_{x\to 1}\left(\frac{1}{x^2+x-2}-\frac{x}{x^3-1}\right)$

26. $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$

27. $\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$

28. $\lim_{x\to a}\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

29. $\lim_{x\to 0}\left(\frac{\sin ax}{\sin bx}\right)$

30. $\lim_{x\to 0}\left(\frac{\sin 5x}{\tan 3x}\right)$

31. $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$

32. $\lim_{x\to 0}\frac{1-\cos 2mx}{1-\cos 2nx}$

33. $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$

34. $\lim_{x\to 0}\frac{\sin 2x+\sin 6x}{\sin 5x-\sin 3x}$

35. $\lim_{x\to 0}\frac{\sec 4x-\sec 2x}{\sec 3x-\sec x}$

36. $\lim_{x\to 0}\frac{\sin x-2\sin 3x+\sin 5x}{x}$

37. $\lim_{x\to 0}\frac{3\sin 2x+2x}{3x+2\tan 3x}$

38. $\lim_{x\to 0}\frac{\sin 3x+7x}{4x+\sin 2x}$

39. $\lim_{x\to 0}\frac{x\tan x}{1-\cos 2x}$

40. $\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3}$

41. $\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}$

42. $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin x^3}$

43. $\lim_{x\to 0}\frac{\sqrt{1+2x}-\sqrt{1-2x}}{\sin x}$

44. $\lim_{x\to 0}\frac{3\sin x-\sin 3x}{x^3}$

45. $\lim_{x\to 0}\frac{3\sin^2 x-2\sin x^2}{3x^2}$

46. $\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$

47. $\lim_{x\to 0}\frac{\tan 3x-2x}{3x-\sin^2 x}$

48. $\lim_{x\to \frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}$

49. $\lim_{x\to \frac{\pi}{4}}\frac{\sin x-\cos x}{x-\frac{\pi}{4}}$

50. $\lim_{x\to \pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}$

51. $\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{(\pi-2x)^2}$
52. $\lim_{x\to\frac{\pi}{4}} \frac{\cos x-\sin x}{\cos 2x}$
53. $\lim_{x\to\frac{\pi}{6}} \frac{\sqrt{3}\sin x-\cos x}{x-\frac{\pi}{6}}$

54. $\lim_{x\to a} \frac{\cos x-\cos a}{x-a}$
55. $\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{x-\frac{\pi}{4}}$
56. $\lim_{x\to\frac{\pi}{6}} \frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$

57. $\lim_{x\to\frac{\pi}{6}} \frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}$
58. $\lim_{x\to 0} \frac{1-\cos x\sqrt{\cos 2x}}{x^2}$
59 $\lim_{x\to 0} \frac{\sin x-2\sin 3x+\sin 5x}{x^3}$

60. $\lim_{h\to 0} \frac{(x+h)^2\sin(x+h)-x^2\sin x}{h}$
61. $\lim_{y\to 0} \frac{(x+y)\sec(x+y)-x\sec x}{y}$

62. $\lim_{x\to\frac{\pi}{6}} \frac{\cot^2 x-3}{\operatorname{cosec} x-2}$
63. $\lim_{x\to\frac{\pi}{2}} (\sec x-\tan x)$
64. $\lim_{x\to\frac{\pi}{4}} \frac{\tan^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$

65. $\lim_{x\to 0}\left(\frac{e^{3x}-e^{-5x}}{x}\right)$
66. $\lim_{x\to 0}\left(\frac{e^{x}+e^{-x}-2}{x^2}\right)$
67. $\lim_{x\to 0}\left(\frac{e^{3x}-1}{x}\right)$

68. $\lim_{x\to 2}\left(\frac{e^{x}-e^{2}}{x-2}\right)$
69. $\lim_{x\to 0}\left(\frac{e^{x}-e^{-x}}{x}\right)$
70. $\lim_{x\to 0}\left(\frac{3^{x}-2^{x}}{\tan x}\right)$

71. $\lim_{x\to 0} \frac{\log(1+2x)}{x}$
72. $\lim_{x\to 0}\left(\frac{3^{2x}-1}{2^{3x}-1}\right)$
73. $\lim_{x\to 0} \frac{x(e^x-1)}{1-\cos x}$

74. $\lim_{x\to 0} \frac{\log(1+x^3)}{\sin^3 x}$
75. $\lim_{x\to 0} \frac{\log(1+ax)-\log(1-bx)}{x}$
76. $\lim_{x\to 0} \frac{(4^x-1)^3}{\sin\frac{x}{4}\log\left(1+\frac{x^2}{3}\right)}$

77. $f(x)=\begin{cases} x^2-1, x\le 1 \\ -x^2-1, x>1 \end{cases}$ Does $\lim_{x\to 1} f(x)$ exist ?

78. Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ where $f(x)=\begin{cases} 2x+3, x\le 0 \\ 3(x+1), x>0 \end{cases}$

79. Find k, so that $\lim_{x\to -1} f(x)$ may exist where $f(x)=\begin{cases} x^3-3x+7, x\le -1 \\ 3x+k, x>-1 \end{cases}$

80. $f(x)=\begin{cases} a+bx, x<1 \\ 4, x=1 \\ b-ax, x>1 \end{cases}$ If $\lim_{x\to 1} f(x)=f(1)$, what are the possible values of a and b?

81. Let f be a function defined by $f(x)=\begin{cases} \frac{x}{|x|}, x\ne 0 \\ 0, x=0 \end{cases}$ Does $\lim_{x\to 0} f(x)$ exist?

82. $f(x)=\begin{cases} \frac{5x}{|x|-2x^2}, x\ne 0 \\ 0, x=0 \end{cases}$ Does $\lim_{x\to 0} f(x)$ exist?

83. $f(x)=\begin{cases}|x|+1, x<0\\ 0, x=0\\ |x|-1, x>0\end{cases}$ For what values of a does $\lim\limits_{x\to a} f(x)$ exist?

84. $f(x)=\begin{cases}1+x^2, 0\le x\le 1\\ 2-x, x>1\end{cases}$ Does $\lim\limits_{x\to 1} f(x)$ exist?

85. $f(x)=\begin{cases}x, 0\le x<\frac{1}{2}\\ 0, x=\frac{1}{2}\\ x-1, \frac{1}{2}<x\le 1\end{cases}$ Show that $\lim\limits_{x\to \frac{1}{2}} f(x)$ does not exist.

86. Evaluate: (i) $\lim\limits_{x\to \frac{5}{2}}[x]$ (ii) $\lim\limits_{x\to 2}[x]$

87. Evaluate $\lim\limits_{x\to 2} f(x)$, where $f(x)=\begin{cases}x-[x], x<2\\ 4, x=2\\ 3x-5, x>2\end{cases}$

88. Find the values of a and b for which both the limits $\lim\limits_{x\to 2} f(x)$ and $\lim\limits_{x\to 3} f(x)$ exist where

$$f(x)=\begin{cases}10+5x, 0\le x\le 2\\ a+bx^2, 2\le x\le 3\\ 3a-bx, 3\le x\le 5\end{cases}$$

89. If $f(x)=\begin{cases}\dfrac{x-|x|}{x}, x\ne 0\\ -2, x=0\end{cases}$ Does $\lim\limits_{x\to 0} f(x)$ exist?

90. Find the value of a for which $\lim\limits_{x\to 0} f(x)$ exist where $f(x)=\begin{cases}a\sin\frac{\pi}{2}(x+1), x\le 0\\ \dfrac{\tan x-\sin x}{x^3}, x>0\end{cases}$

91. Find a relation between a and b so that f defined by $f(x)=\begin{cases}ax+1, x\le 3\\ bx+3, x>3\end{cases}$ is such that

$\lim\limits_{x\to 3} f(x)$ exist.

92. Find a if $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, x<0\\ a, x=0\\ \dfrac{\sqrt{x}}{\left(\sqrt{16+\sqrt{x}}\right)-4}, x>0\end{cases}$ given that $\lim\limits_{x\to 0} f(x)=f(0)$

Derivatives

1. Find the derivative of the following functions w.r.t x using first principle:

(i) x^n (ii) $\sin x$ (iii) $\cos x$ (iv) $\tan x$ (v) $\cot x$ (vi) $\sec x$

(vii) $\cos ec x$ (viii) $\sqrt{2x+3}$ (ix) $\frac{2x+3}{3x+2}$ (x) $x \sin x$ (xi) $x^2 \cos x$

(xii) $\frac{x^2-1}{x^2}$ (xiii) $\frac{x^2+1}{x+1}$ (xiv) $\frac{\tan x}{x}$ (xv) $\cos 3x$

(xvi) $(x-2)(x+1)$ (xvii) $(ax+b)^n$

2. Find the derivatives of the following functions w.r.t x :

(i) $\frac{2x+3}{3x+2}$ (ii) $\frac{x^2+1}{x+1}$ (iii) $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$ (iv) $\frac{3x}{x-1} - \frac{x^3}{2x+1}$

(v) $\sin x \cos x$ (vi) $(5x^3+3x-1)(x-1)$ (vii) $3\cot x + 5\cos ec x$

(viii) $(x+\sec x)(x-\tan x)$ (ix) $(x+\cos x)(x-\tan x)$ (x) $(ax^2+\sin x)(p+q\cos x)$

(xi) $\frac{\sin x+\cos x}{\sin x-\cos x}$ (xii) $\frac{\sin x - x\cos x}{x\sin x+\cos x}$ (xiii) $\frac{x\tan x}{\sec x+\tan x}$ (xiv) $\frac{x^2\cos\left(\frac{\pi}{4}\right)}{\sin x}$

(xv) $x^2 + \frac{4}{x^2} - \frac{2}{3}\tan x + 6e$ (xvi) $\frac{x^5-\cos x}{\sin x}$

(xvii) $\frac{3x^2+2x+5}{\sqrt{x}}$ (xviii) $\frac{x\sin x}{1+\cos x}$ (xix) $\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

3. If $y=\frac{1-\tan x}{1+\tan x}$, show that $\frac{dy}{dx} = \frac{-2}{(1+\sin 2x)}$

Assignment No. 12
Limits and Derivatives

Each part of Q. No.1 & 2 and Q. No. 3- 4 are very short answer type questions:

1. Evaluate the following:

$$(i)\lim_{x\to -2}\left(\frac{1}{x+2}\right)\left(\frac{1}{x}+\frac{1}{2}\right),(ii)\lim_{x\to -1}\left(\frac{x^3+1}{x+1}\right),(iii)\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{4}{x^3-2x^2}\right),(iv)\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$$

$$(v)\lim_{x\to 0}\frac{(8+x)^{\frac{1}{3}}-2}{x}\quad (vi)\lim_{x\to 7}\frac{4-\sqrt{9+x}}{1-\sqrt{8-x}}$$

2. If $f(x)=x^2-x+1$, find $f'(4)$.

3. Differentiate, w. r. to x, the following functions:

$$(i)(x-a)(x^2-b),(ii)x^3\tan x,(iii)\frac{x+3}{x^2+1}.$$

4. If for the function f,given by $f(x)=kx^2+7x-4, f'(5)=97$, find k.

5. Evaluate the following limits:

$$(i)\lim_{x\to 2}\frac{x^3+3x^2-9x-2}{x^3-x-6},(ii)\lim_{x\to 0}\frac{x\cos x+\sin x}{x^2+\tan x},(iii)\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3},$$

$$(iv)\lim_{x\to 0}\frac{1-\cos 4x}{1-\cos 6x},(v)\lim_{x\to \pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}.$$

$$(vi)\lim_{x\to \frac{\pi}{4}}\frac{\sqrt{2}-\cos x-\sin x}{(4x-\pi)^2}$$

6. Differentiate the following functions w. r. to x from the first principles:

$$(i)x-\frac{1}{x},(ii)x^2+\frac{1}{x^2},(iii)sec(x+1),(iv)\frac{\sin x}{x},(v)x\cos x.$$

7. Differentiate, w. r. to x, the following functions:

$$(i)\frac{\sec x-1}{\sec x+1},(ii)\frac{x\sin x}{1+\cos x},(iii)\left(x-\frac{1}{x}\right)\left(x^2-\frac{1}{x^2}\right),(vi)\frac{2}{x+1}-\frac{x^2}{3x-1}.$$

8. Evaluate the following limits, if they exist:

$$(i)\lim_{x\to 0} f(x) \text{ where } f(x)=\begin{cases}\dfrac{3x}{|x|+2x}, x\neq 0\\ 0, x=0\end{cases},$$

$$(ii)\lim_{x\to \frac{1}{2}} f(x) \text{ where } f(x)=\begin{cases}x, 0\leq x<\dfrac{1}{2}\\ \dfrac{1}{2}, x=\dfrac{1}{2}\\ 1-x, \dfrac{1}{2}<x\leq 1\end{cases},$$

$$(iii)\lim_{x\to 2} f(x) \text{ where } f(x)=\begin{cases}x-[x], x<2\\ 4, x=2\\ 3x-5, x>2\end{cases}.$$

9. Let $f(x)=\begin{cases}ax-4, x<1\\ 1, x=1\\ 4x^2+bx, x>1\end{cases}$ and if $\lim_{x\to 1} f(x)=f(1)$, what are the possible values of a and b?

10. Evaluate the following limits:

$$(i)\lim_{x\to 0}\frac{e^{2x}-2e^{x}+1}{x^2}, (ii)\lim_{x\to 0}\frac{e^{5x}-e^{3x}}{x}, (iii)\lim_{x\to 0}\frac{\log(1+5x)}{\sin x}, (iv)\lim_{x\to 0}\frac{6^x-2^x-3^x+1}{x^2}$$

Learning Objectives
At the end of this unit the student will be able to: • define Limit of a function. • determine the condition for existence of limits. • derive limits formulae and evaluate limits. • derivative of a function at a point from the first principle. • derivatives of various functions from the first principles. • establish various rules for differentiation. • differentiate various functions using rules for differentiation.

Probability

Definitions:

1. Random Experiment: When a coin is tossed it may turn up a head or a tail, but we are not sure which one of these results will actually be obtained. Such experiments are called random experiments. An experiment is called a random experiment if it has more than one possible outcome and if it is not possible to predict the outcome in advance.
2. Outcomes and Sample space: A possible result of a random experiment is called its outcome. The set of outcomes is called the sample space. For example in the experiment of tossing a die the outcomes are 1, 2, 3, 4, 5 and 6. The sample space is {1, 2, 3, 4, 5, 6}. Each element of the sample space is called a sample point.
3. Event: Subset E of a sample space is called an event. For example in the tossing of two coins the sample space is {HT, TH, HH, TT}. Event E is the occurrence of exactly one head. The event E {HT,TH} is the subset of the given sample space.
4. Occurrence of event: Consider the experiment of throwing of a die. Let the event be `a number less than 3'. If on throwing the die we get a 2, then we say that the event has occurred.
5. Types of events:

a) Impossible event: $\emptyset \subseteq S$ is an impossible event, as it contains no elements and hence cannot take place. For example E = getting a number less than 0 in a single throw of a die is an impossible event.

b) Sure event: $S \subseteq S$ is sure event as it always occurs. For example E = getting a number greater than 0 in a throw of a die is a sure event.

c) Simple event or an elementary event: Event containing one sample point are called simple event. For example in tossing of a coin {H}, {T} are simple events.

d) Compound event: Event containing more than one sample point is called a compound event. For example in tossing a coin twice, the event of getting exactly one head is a compound event.

e) Mutually exclusive events: Two or more events are said to be mutually exclusive if the occurrence of one restricts the occurrence of the other. If A and B are mutually exclusive then $A \cap B = \emptyset$.

f) Exhaustive events: Events A, B, C,……..,H of S are said to be exhaustive if $A \cup B \cup C \cup \dots \dots . \cup H = S$.

g) Mutually exclusive and Exhaustive events:

h) Equally likely events: Events are said to be equally likely, if under the given circumstances, we cannot prefer one event to other event. For example, in tossing a coin, the coming up of the head or the tail is equally likely.

6. Algebra of events: Let A and B be two events of S. Then:

i. $A \cup B$ = event that either A or B or both occur.
   = event that at least one of them occurs.

ii. $A \cap B$ = event that both A and B occur simultaneously.

iii. $A' or\ \overline{A}$ = complementary event is an event that A does not occur.

Classical definition of probability: If an experiment results in `n' mutually exclusive, equally likely and exhaustive outcomes out of which `m' outcomes are favourable to the occurrence of an event E, then the probability of occurrence of E, denoted by P(E) is given by:

$$P(E) = \frac{no.\ of\ outcomes\ favourable\ to\ E}{no. of\ possible\ outcomes} = \frac{m}{n} \Rightarrow P(E) = \frac{n(E)}{n(S)}$$

Probability of non-occurrence of an event: Probability of non-occurrence of an event E is given by:

$P(\overline{E}) = \frac{n(\overline{E})}{n(S)} = \frac{n(S)-n(E)}{n(S)} = 1 - P(E)$. Therefore, $P(E) + P(\overline{E}) = 1$.

Results on Probability

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. If A and B are mutually exclusive then, $A \cap B = \emptyset \Longrightarrow P(A \cap B) = 0$. Hence if A and B are mutually exclusive $P(A \cup B) = P(A) + P(B)$.
3. If A and B are mutually exclusive and exhaustive events, then $P(A) + P(B) = 1$.
4. Since $A\ and\ \overline{A}$ are mutually exclusive and exhaustive events $P(A) + P(\overline{A}) = 1$.
5. If A and B ⊆ S, then the following events are:
   a. Only A occurs= $A \cap \overline{B} = A - B$
   b. Only B occurs= $\overline{A} \cap B = B - A$
   c. Either A or B occurs = $A \cup B$
   d. Neither A nor B occurs = Not A and Not B = $\overline{A} \cap \overline{B}$ = $(\overline{A \cup B})$ (by De Morgan's Law)
6. P(Neither A nor B occurs) = $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 -$ P$(A \cup B) = 1 - P(A\ or\ B)$
7. P(only A occurs) = $P(A - B) = P(A) - P(A \cap B)$.
   Similarly, P(only B occurs) = $P(B - A) = P(B) - P(A \cap B)$

Exercise

1. A die is thrown. If the die shows an even number, a coin is thrown. If the coin shows a head, a ball is drawn from a bag containing 2 red and 1 black ball. Find the sample space of this experiment. Find also the subset of the event when 1 red ball is selected.
2. A coin is tossed. If the result is a tail, the coin is tossed again. Otherwise a die is rolled. Describe the following events:
   a. Getting at least one tail
   b. Getting a head
   c. Getting a head and an even number
   d. Getting an odd number

3. An experiment involves tossing a coin and rolling a die recording the outcomes that come up. Describe the following events:
   A: getting a head and even number
   B : getting a prime number
   C : getting a tail and an odd number
   D : getting a head or tail.
   Which pairs of the events are mutually exclusive?
4. Two dice are thrown. The events A, B, C and D are given below:
   A: getting an odd number on the first die
   B: getting an even number on the second die.
   C: getting the product on the dice ≥ 20
   D: getting the product on the dice between 15 and 24

a. Describe the events $\overline{A}, B\ or\ C, A\ and\ D$.
b. Are A and B mutually exclusive; C and D mutually exclusive; B and C mutually exclusive and exhaustive?
c. Is $A = \overline{B}$?

5. In a single throw of two dice, what is the probability of getting a) doublet b) a total of more than 10 c) an odd number on one and a multiple of 3 on the other d) a multiple of 2 on one and a multiple of 3 on the other e) the sum as a prime number.
6. In a single throw of three dice, determine the probability of a) getting a total of 5 b) a total of at most 5.
7. A bag contains 6 white and 4 red balls. 3 balls are drawn from the bag. Find the probability that a) all the balls are white b) 2 white and 1 red ball is drawn.
8. A bag contains 20 tickets bearing 1 to 20. Two tickets are drawn from a bag. Find the probability that both numbers on the tickets are prime.
9. Find the probability that in a random arrangement of the letters of the word `COURSE' all the vowels do not come together.
10. Find the probability that in a random arrangement of the letters of the word `VARANASI', the three A's come together.
11. Two unbiased dice are thrown. Find the probability of getting a total of a) 5 b) either 9 or 11 c) neither 9 nor 11
12. A five-digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number is divisible by 4.
13. Two unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear.
14. A card is drawn from a deck of cards. Find the probability of getting a king or a heart or a red card.
15. A box contains cards numbered 1 to 100. A card is drawn. Find the probability that the number on the card is a multiple of 5 or 8.
16. A bag contains 3 red, 4 black and 2 green balls. Two balls are drawn at random from the bag. Find the probability that both the balls are of different colours.
17. Two cards are drawn from a well-shuffled pack of 52 cards without replacement. Find the probability that neither a jack nor a card of spade is drawn.
18. Two students A and B appeared in an examination. The probability that A will qualify the examination is 0.05 and that B will qualify the examination is 0.10. The probability that both will qualify the examination is 0.02. Find the probability that a) both A and B will not qualify the examination b) at least one of them will not qualify the examination c) only one of them will qualify the examination.
19. Find the probability that when a hand of 7 cards is drawn from a well-shuffled deck of 52 cards, it has a) all kings b) 3 kings c) at least 3 kings.
20. An integer is chosen at random from the first 200 positive integers. Find the probability that the integer is divisible by 6 or 8.
21. Probability that Salma passes in Mathematics is $\frac{2}{3}$ and the probability she passes in English is $\frac{4}{9}$. If the probability of passing both courses is $\frac{1}{4}$, what is the probability that she will pass in at least one of these subjects.
22. Six boys and six girls sit in a row at random. Find the probability that the six girls sit together. Also find the probability that boys and girls sit alternately.
23. Out of 9 outstanding students in a college, there are 4 boys and 5 girls. A team of four students is to be selected for a quiz programme. Find the probability that two are boys and two are girls.

24. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random find the probability that a) both the balls are red b) one ball is white c) the balls are of same colour d) one is white the other is red.
25. Four cards are drawn at random from a pack of 52 cards. Find the probability of getting a) all the four cards of the same suit b) all the four cards of the same number c) one card from each suit d) two red cards and two black cards e) all cards of the same colour f) all face cards.
26. A class contains 10 boys and 8 girls. Three students are selected at random. What is the probability that the selected group has a) all boys b) all girls?
27. The probability that at least one of A and B occur is 0.6. If A and B occur simultaneously with a probability 0.2. Find $P(\overline{A}) + P(\overline{B})$.
28. $P(A \cap B) = \frac{1}{2}$, $P(A' \cap B') = \frac{1}{3}$, $P(A) = p, P(B) = 2p$, then find the value of p.
29. If $P(A) = \frac{1}{2}, P(B) = p, P(A \cup B) = \frac{3}{5}$, then find p, if A and B are mutually exclusive.
30. A committee of 5 principals is to be selected from a group of 6 male principals and 8 female principals. If the section is made randomly, find the probability that there are 3 female principals and two male principals.

Assignment No. 13
Probability

1. An experiment involves rolling a pair of dice and recording the numbers that come up. Consider the following events:

A = The sum is even
B = The sum is a multiple of 3
C = The sum is less than 4
D = The sum is greater than 11

Prove that

(i) A and B are not mutually exclusive.

(ii) A and C are not mutually exclusive.

(iii) A and D are not mutually exclusive.

(iv) B and C are not mutually exclusive.

(v) B and D are not mutually exclusive.

(vi) C and D are mutually exclusive.

2. Find the probability that in a random arrangement of the letters of the word REPUBLIC, the vowels do not come together.

3. An urn contains 9 red, 7 white and and 4 black balls. If two balls are drawn at random, find the probability that the balls are of the same colour.

4. The probability that a person visiting a dentist will have his teeth cleaned is 0.44, the probability that he will have a cavity filled is 0.24. The probability that he will have his teeth cleaned or a cavity filled is 0.60. What is the probability that a person visiting the dentist will have his teeth cleaned and cavity filled?

5. In a single throw of two dice, find the probability that neither a doublet nor a total of 9 will appear.

6. Of the students attending a lecture, 50% could not see what was written on the board and 40% could not hear what the lecture was saying. Most unfortunate 30% fell into both of these categories. What is the probability that a student picked at random was able to see and hear satisfactorily?

7. In a class of 100 students, 60 drink tea, 50 drink coffee and 30 drink both. A student from this class is selected at random. Find the probability that the student takes (i) at least one of two drinks. (ii) only one of two drinks.

8. A basket contains 20 apples and 10 oranges out of which 5 apples and 3 oranges are defective. If a person takes out 2 at random, what is the probability that either both are apples or both are good?

9. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is (i) divisible by 3 or 4? (ii) neither divisible by 3 nor 4? (iii) either a multiple of 3 or 2?

10. 4 cards are drawn from a well-shuffled deck of cards. What is the probability of obtaining 3 diamonds and one spade?

11. The number lock of a suitcase has 4 wheels, each labeled with ten digits i.e. 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

12. Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that the envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Learning Objectives
At the end of this chapter the student will be able to: • define the Random Experiment, outcomes, sample space. • define Events, Occurrence of events, “not”, “and”, “or” event. • define Probability of an Event. • apply the concept to solve simple problems.

QUESTION BANK

Q. No. 1- 42 are very short answer type questions:

1.Write the set $\left\{\frac{2}{2},\frac{4}{3},\frac{6}{4},\frac{8}{5},\frac{10}{6},\frac{12}{7},\frac{14}{8}\right\}$ in set builder form.

2.Find the domain of the real valued function $f(x)=\frac{1}{x^2-3x+2}$

3.Let $n(A)=2$ & $n(B)=3$, then find the number of relations from A to B.

4.Write the relation $R=\left\{(x,x^3): x \text{ is a prime number less than } 6\right\}$ in roster form.

5.Find the domain and the range of the relation $\{(x,y): x\in N, x+y=5\}$

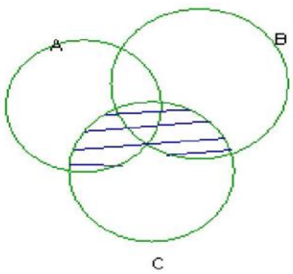
6. If $A=\{1,2,3\}$ & $B=\{4,5,6\}$, is $R=\{(4,2),(1,4),(1,6)\}$ a relation from A to B. Justify.

7.Write the value of $2\cos^2 15^\circ-1$.

8.Prove that $\frac{1-\cos A}{1+\cos A}=\tan^2\frac{A}{2}$

9.Solve $5x-3<3x+1$ when (i) x is an integer (ii) x is a real number

10.What does the shaded portion in the figure represent?



11.If the ordered pairs $(a,-1)$ and $(5,b)$ belong to $\{(x,y): y=2x-3\}$. Find the values of a and b.

12.Find the real values of x and y if : $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$

13.Express complex number $\frac{i^2+i^4+i^6+i^7}{1+i^2+i^3}$ in the form $a+ib$

14.Find the mirror image of point $(-7,2,-1)$ in the ZX -plane.

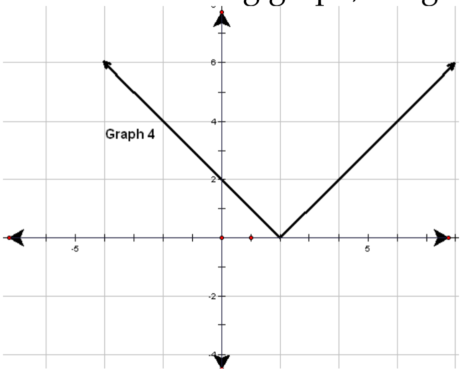
15.Find y if the slope of the line joining $(-8,11),(2,y)$ is $\frac{-4}{3}$.

16.Find the distance of the point $P(-1,1)$, from the line $12(x+6)=5(y-2)$.

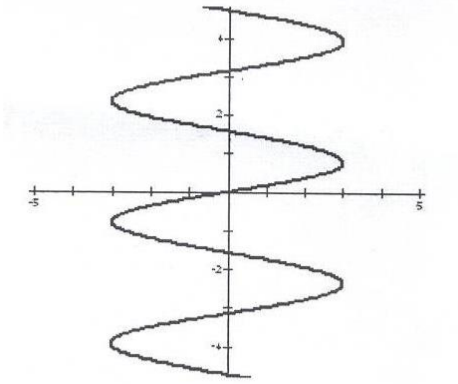
17.For the parabola $x^2=-9y$,find the length of the latus rectum

18.Find the center and radius of the circle $x^2+y^2-4x-8y-45=0$

19.Is the following graph, the graph of a function of x? Justify



20.Is the following graph, the graph of a function of x? Justify



21.Find the 7th term in the expansion of $\left(x^2+\frac{2}{x}\right)^9$.

22.Write down the general term in the expansion of $\left(\frac{1}{x}-x^3\right)^{10}$.

23.Find the coefficient of a^5 in the expansion of $(a+5)^6$.

24.Find the 4th term from the end in the expansion of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^9$.

25.Find the sum of all numbers with two digits.

26.If A is the A. M. between a and b, prove that $(A-a)^2+(A-b)^2=\frac{1}{2}(a-b)^2$.

27.If G is the G. M. between two distinct positive numbers a and b, then show that $\frac{1}{G-a}+\frac{1}{G-b}=\frac{1}{G}$.

28.Evaluate $\sum_{1}^{10}\left(2^n-1\right)$.

29.Find the nth term of the series: $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+....$

30.Find the sum: $5^3+6^3+7^3+....+10^8$.

31.If $(n+1)!=12(n-1)!$, find n.

32.Prove that $n!+(n+1)!=(n+2)n!$

33.How many different numbers of three digits can be formed without using the digits 0, 2, 3, 4, 5 and 6?

34.Two cards are drawn from a deck of 52 cards one by one with replacement. Find the number of ways in which it can be done.

35. There are 12 doors in a hall. In how many ways can a person enter the hall through a door and leave it by a different door?

36.Evaluate: $\lim_{x\to -1^{+}} \sqrt{x+1}$. What can you say about $\lim_{x\to -1^{-}} \sqrt{x+1}$?

37.Evaluate the following limit: $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$

38. If $f(x)=2x^2+1$, find $f'(0)$.

39.Differentiate, w. r. to x, the following function: $2\sin x+5x^2\cot x$.

40.If A and B are two mutually exclusive events.

If $P(A)=\frac{1}{4}, P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{1}{2}$. Find the value of $P\left(A\cap \overline{B}\right)$.

41.The probabilities that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, find $P\left(\overline{A}\right)+P\left(\overline{B}\right)$.

42. If $f(x)=x^3-\frac{1}{x^3}$, then find the value of $f(x)+f\left(\frac{1}{x}\right)$

43. Let $U=\{1,2,3,4,.........10\}$, $A=\{1, 2, 3, 4, 5\}$, $B=\{1, 2, 3, 4, 6, 8, 10\}$

$C=\{1,3,5,7,9\}$, find (a) $(A\cap B)'$ (b) $(B-C)'$ (c) $(A\cup B)'-(A\cup C)'$

44.Prove the following:

$(i)\ \frac{\cos 5x+\cos 3x}{\sin 5x-\sin 3x}=\cot x.$

$(ii)\ \cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)=\frac{3}{2}$

(iii) $\frac{\sin 8x\cos x-\sin 6x\cos 3x}{\cos 2x\cos x-\sin 3x\sin 4x}=\tan 2x$

45. $\tan A=\frac{3}{4}, \cos B=\frac{9}{11}$, where $\pi<A<\frac{3\pi}{2}$ and $0<B<\frac{\pi}{2}$, then find the value of $\tan(A+B)$.

46.Find the equations of the lines which passes through the point $(3,4)$ and the sum of its intercepts on the axes is 14

47.If origin is the centroid of the triangle PQR with vertices $P(2a,2,6)$, $Q(-4,3b,-10)$ and $R(8,14,2c)$, then find the values of a,b,c

48.Find the equation of the line midway between parallel lines $9x+6y-7=0$ and $3x+2y+6=0$

49.Simplify each of the following:

(i) $\dfrac{\sin(180^\circ+\theta)\cos(360^\circ-\theta)\tan(270^\circ-\theta)}{\sec^2(90^\circ+\theta)\tan(-\theta)\sin(270^\circ-\theta)}$.

(ii) $\dfrac{\cos(2\pi+\theta)\operatorname{cosec}(2\pi+\theta)\tan\left(\frac{\pi}{2}+\theta\right)}{\sec\left(\frac{\pi}{2}+\theta\right)\cos\theta\cot(\pi+\theta)}$.

50.Prove that: (i) $\cos A\cos(60^\circ-A)\cos\left(60^\circ+A\right)=\frac{1}{4}\cos 3A.$

(ii) $1+\cos^2 2x=2\left(\cos^4 x+\sin^4 x\right)$

51.Find the general solution:

(i) $\tan x+\sec x=\sqrt{3}$

(ii) $\cos 3x+\cos x-2\cos 2x=0$

(iii) $4\cos^2 x-4\sin x-1=0$

52.In a town of 10000 families, it was found that 4000 families read newspaper A,2000 families read newspaper B, 1000 families read newspaper C, 500 families read both A and B, 300 families read both B and C and 400 read both A and C. 4000 families read neither A nor B nor C. Find the number of families that read (i) all the three (ii) exactly two newspapers (iii) A and C but not B (iv) do not read A.

53.Solve the following quadratic equations:

(i) $3x^2-4x+\frac{20}{3}=0$ (ii) $\sqrt{5}x^2+x+\sqrt{5}=0$

54. If $Z_1=2-i$ and $Z_2=-2+i$, find (i) $\operatorname{Re}\left(\frac{Z_1Z_2}{\overline{Z_1}}\right)$ (ii) $\operatorname{Im}\left(\frac{1}{Z_1\overline{Z_1}}\right)$

55.Find the modulus of $\dfrac{2+i}{4i+(1+i)^2}$

56.Convert the complex number $3\left(\cos\frac{5\pi}{3}-i\sin\frac{\pi}{6}\right)$ into polar form.

57.Find the equation of the circle whose centre is $(2,-3)$ and passes through the intersection of the lines $3x-2y-1=0$ and $4x+y-27=0$

58.Find the equation of the ellipse with eccentricity $\frac{3}{4}$, foci on y-axis, center at the origin and passing through the point $(6,4)$.

59.Find the coordinates of the foci, the vertices , the eccentricity and the length of the latus rectum of the hyperbola: $5y^2-9x^2=36$

60.Find the equation of the line on which the perpendicular from origin makes an angle of $30°$ with the x-axis and which forms a triangle of area $\frac{50}{\sqrt{3}}$ square units with the axes

61.One diagonal of a square lies along the line $x-2y+2=0$ and one vertex of the square is $(1,4)$. Find the equations of all the sides and the other diagonal of the square.

62.If the image of the point $(2,1)$ in a line is $(4,3)$, find the equation of the line.

63.A ray of light is sent along the line $x-2y-3=0$. Upon reaching the line $3x-2y-5=0$ the ray is reflected from it. Find the equation of the line containing the reflected ray.

64.If the points $P(0,-11,4), Q(4,p,-2)$ and $R(2,-3,1)$ are collinear, find the value of p.

65.Find the domain and range of $f(x)=\frac{1}{\sqrt{4-x^2}}$.

66.Find the equation of the circle that passes through $(2,-2),(3,4)$ and has center on the line $2x+2y=7$.Find the center and radius.

67.Find the equation of the line passing through the intersection of the lines $3x+y-9=0$ and $4x+3y-7=0$ and perpendicular to the line $5x-4y+1=0$

68.Solve the following system of linear inequalities, and represent the solution(if it exists) on the number line:

$$2(2x+3)-10<6(x-2), \frac{2x-3}{4}+6\geq 4+\frac{4x}{3}$$

69.Solve the following system of linear inequalities graphically:

$3y-2x\leq 4, x+3y>3, x+y\geq 5, y<4$

$x+y<5, 4x+y\geq 4, x+5y\geq 5, x\leq 4, y\leq 3$

70.Write the following complex numbers in polar form:

(i) $\frac{5-i}{2-3i}$ (ii) $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

71. If $\sin x=-\frac{\sqrt{5}}{3}$, x lies in the third quadrant,then find thevalue of $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$.

72. Find the coefficient of x^{-2} in the expansion of $\left(x+\frac{1}{x^3}\right)^{11}$.

73.Find the term independent of x in the expansion of: $\left(x^2-\frac{2}{x^3}\right)^5$.

74.Find the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ and that of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ and then find the relation between a and b so that these coefficients are equal.

75.Find the middle terms in the expansion of $\left(3x-\frac{x^3}{6}\right)^7$.

76.If the coefficients of the three successive terms in the expansion of $\left(1+x\right)^n$ are in the ratio 1: 7: 42, find n.

77.If in the expansion of $(1-x)^{2n-1}$,the coefficient of x^r is denoted by a_r, then prove that $a_{r-1}+a_{2n-r}=0.$

78.If the 3rd, 4th, 5th and 6th terms in the expansion of $(x+a)^n$ respectively are a,b,c and d. Prove that $\dfrac{b^2-ac}{c^2-bd}=\dfrac{5a}{3c}$.

79.Prove that the term independent of x in the expansion of $\left(1+x^m\right)\left(1+\dfrac{1}{x}\right)^n$ is ${}^{m+n}C_n$.

80.If a,b,c are in A. P., prove that $a\left(\dfrac{1}{b}+\dfrac{1}{c}\right),b\left(\dfrac{1}{c}+\dfrac{1}{a}\right),c\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ are also in A. P.

81.If S_1,S_2,S_3 be the sums of n terms, $2n$ terms and $3n$ terms respectively of a G.P., then prove that $S_1^{\,2}+S_2^{\,2}=S_1\left(S_2+S_3\right)$.

82.If $(b-c)^2,(c-a)^2,(a-b)^2$ are in A. P., prove that $\dfrac{1}{b-c},\dfrac{1}{c-b},\dfrac{1}{a-b}$ are also in A. P.

83.Find four numbers in G. P. whose sum is 85 and the product is 4096.

84.If $\dfrac{1}{a+b},\dfrac{1}{2b},\dfrac{1}{b+c}$ are three consecutive terms of an A. P., prove that a,b,c are three consecutive terms of a G. P.

85.If a,b,c,d are in G. P., show that a + b, b + c, c + d are also in G. P.

86.How many 7 digit numbers can be formed using the digits 1, 2, 2, 0, 4, 2, 4?

87.If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, prove that the word SACHIN appears at serial number 601.

88.A hockey team of 11 players is to be selected from two groups of 6 and 10 respectively. In how many ways can the selection be made if the group of 6 shall contribute at least 4 players?

89.Evaluate the following limit, if it exists: $\lim\limits_{x\to 0} f(x)$ where $f(x)=\begin{cases}\dfrac{x}{|x|+x^2}, x\neq 0\\ 0, x=0\end{cases}$.

90.Evaluate the following limit, if it exists: $\lim\limits_{x\to 5} f(x)$, where $f(x)=[x+3]-\dfrac{2|x-5|}{x-5}+4x^2$.

91.Differentiate w. r. to x, the following functions: $(i)\dfrac{x\cos x+\cot x}{3x+5},(ii)\left(x-\dfrac{1}{x+1}\right)\left(3x^2\right)$

92.Evaluate the following limit (if it exists): $\lim\limits_{x\to 0} f(x)$, where $f(x)=\begin{cases}2x-1, x<o\\ o, x=0\\ x^2+1, x>0\end{cases}$

93.Evaluate the following limit (if it exists): $\lim\limits_{x\to 0} f(x)$, where $f(x)=\begin{cases}[x], x>o\\ o, x=0\\ x^2, x<0\end{cases}$

94.Evaluate the following limit: $\lim_{y \to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y.}$

95.There are three red and three black balls in a bag. 3 balls are taken out at random from the bag. Find the probability of getting 2 red and 1 black balls or 1 red and 2 black balls.

96. Find the probability of 4 turning up for at least once in two tosses of a fair die.

97.A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one of them are chosen at random, what is the probability that it is rusted or a bolt?

98.A box contains 100 bolts and 50 nuts. It is given that 50% bolts and 50% nuts are rusted. Two objects are selected from the box at random. Find the probability that both are bolts or both are rusted.

99.The probability that a patient visiting a dentist will have a tooth extracted is 0.06, the probability that he will have a cavity filled is 0.2 and the probability that he will have a tooth extracted as well as cavity filled is 0.03. What is the probability that a patient has either a tooth extracted, or a cavity filled?

100.A card is drawn from a well shuffled pack of 52 cards. Find the probability that the card is either a jack or a king or a queen.

Do’s and Don’ts for making a Presentation

When used effectively, Google Slides/ PowerPoint/Prezi are powerful tools which can help you create professional presentations. However, it is worth reminding ourselves of some basic dos and don'ts when designing slides.

Get to the point

Try not to put too much of your presentation script in your slide show. It's not a good idea to use lots of text over too many lines; this makes your slides look cramped, as well as being difficult to follow. If you do have too much text on a slide, there is the danger that you'll be tempted to start reading from the screen rather than communicate with your audience. This makes it difficult to engage or interact with them.

Do not make your audience read the slide rather than listen to what you are saying. The slides should support what you're saying - not say it all for you. The text on the slides should be used as prompts or to back up your messages. Try not to let one point run for more than two lines. A good guide is if a point has lots of punctuation, you are probably trying to say too much. Do make sure that there's lots of white space on your slide, so that text doesn't look cramped or cluttered.

Special effects

Do not confuse your audience by having text and images appearing from the left, right, top, bottom and diagonally on a slide. When used selectively, animation features can be very effective. Do use the odd animated effect, but consider if your presentation really needs it. Keep to a simple style to present your text and retain the same effect throughout your presentation.

Colour codes

Your slides will be very difficult to read if you use too much colour, and they'll also look less professional. Do choose a background colour that's easy on the eye, and make sure your text colour is a suitable contrast. Dark colours on a light background work well. Softwares have tools to ensure that you always pick complementary colour schemes to create a professional look and feel.

Text size

Do make sure the size of your slide headings doesn't dominate the rest of your text. Don't use large text (eg 72 points) with much smaller body text (eg 20 point), as it will look mismatched. At the same time, you need to make sure your text is large enough to read on screen - think of the people viewing from the back of the room. A point size of 20 or above is a good size to ensure your audience can comfortably read the text, with headings set in a larger size.

Don’t use fonts with serifs (thin lines) like Times New Roman. Fonts without serifs, like Arial, are easier to read. Don’t mix fonts within your presentation - a lack of consistency

looks un-professional. Use left justification - it is easier to read than centered, right or fully justified text
(both edges). Words/paragraphs in capital letters, italics or underlined are harder to read.

Best use of images

If you are going to use images, make sure they're appropriate to the points you're trying to make and don't place images on the slide so that they overshadow everything else. Remember to give credit to the source from where the image is taken

Transition slides

Don't make your audience feel uncomfortable by selecting one of the more outlandish transition styles to move from slide to slide - especially if you opt for a different style each time. For a standard presentation, do use a transition effect that is unobtrusive and subtle. The effect transition slides should only be used if you are trying to make a point.

Is your layout clear?

Do chose one layout style for every slide, such as a main heading with bullet points underneath - it's easy to read and follow. Take advantage of the Themes and Quick Styles available in PowerPoint 2007 to ensure a professional looking layout that has continuity with colour and type face.

Don't be caught out - preview your slide show to ensure you know the final content of each slide.

Charts, graphs and diagrams

Do use the tools to add charts, graphs and diagrams into your presentations, but keep these straightforward and to the point. The SmartArt tool can be used to help present complex information in a simple, easy to understand way. It's a good idea to ensure that these elements are properly labelled with a reference so that people can understand their relevance.

CASE STUDY

Scan the QR CODE to access the Google forms:

CASE STUDY 1: Permutations and Combinations

CASE STUDY 2: Limits

CASE STUDY 3: Derivatives

ANSWERS OF THE ASSIGNMENTS

ASSIGNMENT NO. 1 (SETS)

1. $\left\{x: x = \frac{n}{n^3+1}, n \leq 6, n\epsilon N\right\}$
2. $\left\{x: x = \frac{n}{n+2}, n \text{ is an odd natural number} \leq 11\right\}$
3. $\{0, \pm1, \pm2, \pm3, \pm4, \pm5\}$
4. $\{17, 26, 35, 44, 53, 62, 71, 80\}$
5. $A = C$; $A and C$, $D and E$ are equivalent
6. (a) finite (b) infinite (c) empty (d) infinite (e) empty (f) finite
7. $A \cap B = \{x : x = 15n, n\epsilon Z\}$
8. $P(A) = \{\emptyset, \{1\}, \{2\}, \{\{3,4\}\}, \{1,2\}, \{1,\{3,4\}\}, \{2,\{3,4\}\}, A\}$

10. False (i) , (ii) ,(iii) , (vi) , (viii) True (iv) , (v) , (vii)
12. (i) $\{6,8\}$ (ii) $\{1,3,5,7,9\}$ (iii) $\{1\}$ (iv) $\{3,5,6,7,8\}$
13. (i) 7 (ii) 6 (iii) 10

14. (i) 18 (ii) 3
15. (i) 62 (ii) 39 (iii) 1
16. (i) 35 (ii) 11 (iii) 11

ASSIGNMENT NO. 2 (RELATIONS AND FUNCTIONS)

1. $\{(2,5), (3,5)\}$
2. Not a function
3. Yes it's a function
4. 2^4
5. $Domain = \{-3, -2, -1, 0, 1, 2, 3\}$, Range = $\{0, 1, 2, 3, 4\}$

7. D = $\{2, 3, 4, 5, 6, 7\}$ $R = \{0, 1, 2, 3, 4, 5\}$
8. Not a function

9. $R = \{-1\}$
10. ±4
11. $\pm\frac{1}{\sqrt{2}}$
12. $R = \{(2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (5,5), (6,6)\}$
13. Not equal because their domains are not equal
14. (i) $D_f = R$, $R_f = (-\infty, 1]$ (ii) $D_f = R - \{\pm\sqrt{2}\}$, $R_f = (-\infty, 0) \cup \left[\frac{3}{2}, \infty\right)$
    (iii) $D_f = [-3,3]$, $R_f = [0,3]$ (iv) $D_f = (3, \infty)$, $R_f = (0, \infty)$
15. $(f+g)(x) = \begin{cases} 2x, x \geq 0 \\ 0, x < 0 \end{cases}$, $(f-g)(x) = \begin{cases} 0, x \geq 0 \\ 2x, x < 0 \end{cases}$,
    $(fg)(x) = \begin{cases} x^2, x \geq 0 \\ -x^2, x < 0 \end{cases}$, $\left(\frac{f}{g}\right)(x) = \begin{cases} 1, x > 0 \\ -1, x < 0 \end{cases}$
16. $(i)(g-f)(x) = -4 + 8x, D_{g-f} = R$ $(ii)\frac{f}{g}(x) = \left(\frac{x-1}{x}\right)^2, D_{\frac{f}{g}} = R - \{0\}$
    (iii)$8x^2 - 18x + 8, x \geq 0; 8x^2 - 14x + 8, x < 0$
    $(iv)\frac{4}{g}(x) = \frac{1}{x^2}, D_{\frac{4}{g}} = R - \{0\}$ 17. -4

ASSIGNMENT NO. 3 (TRIGONOMETRIC FUNCTIONS)

(1).$18^{\circ}19'38''$ (2) $-\frac{2\sqrt{3}}{3}$ (4) $\theta = 2n\pi \pm \frac{2\pi}{3}; n \in Z$

(5) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (7)$2+\sqrt{3}$ (9) $\frac{3}{5}$, $\frac{117}{44}$ (11)(i) $\frac{n\pi}{4}$ or $n\pi \pm \frac{\pi}{3}; n \in Z$

(ii) $2n\pi - \frac{\pi}{4}; n \in Z$ or $n\pi + (-1)^n\left(-\frac{\pi}{2}\right) + \frac{\pi}{4}$; $n \in Z$ $(iii) n\pi + (-1)^{n+1}\frac{\pi}{6}$ $or\ m\pi + (-1)^{m+1}\frac{\pi}{2}, m, n \in Z$

$(iv)\frac{2n\pi}{5} - \frac{\pi}{10}$ $or\ 2m\pi - \frac{\pi}{2}, m, n \in Z$ (13) $\frac{2\sqrt{5}}{5}$, $-\frac{\sqrt{5}}{5}$, -2

ASSIGNMENT NO. 4 (LINEAR INEQUALITIES)

1. (i)
2. $x \geq \frac{5}{2}$
3. $\{......., -2, -1, 0, 1, 2, 3\}$
4. $(-2, 5)$
5. $62.5 < x < 400$
6. i) {1,2} ii) {……..,0,1,2} iii) $(-\infty, 5/2)$
7. 8 and 10; 10 and 12; 12 and 14;
8. $6.27 < x < 8.07$
9. (i) $x < -\frac{13}{2}$ (ii) $x < 1$ or $x > 11/3$ (iii) $x < 0$ or $x > 4$ (iv) $\frac{19}{18} \leq x \leq \frac{29}{18}$ (v) $x > \frac{5}{2}$
10. (i) $x \leq 2$ (ii) $x > \frac{40}{11}$

ASSIGNMENT No. 5 (COMPLEX NUMBERS AND QUADRATIC EQUATIONS)

1. $x = 1/6$, $y = 5/2$
2. $6 - 4i$
3. 0
4. 2
5. $\frac{2}{13} - \frac{3}{13}i$
6. $(i) 2\sqrt{2}\left[cos\left(-\frac{3\pi}{4}\right) + isin\left(-\frac{3\pi}{4}\right)\right]$ (ii) $\frac{1}{\sqrt{2}}\left[cos\left(-\frac{\pi}{4}\right) + isin\left(-\frac{\pi}{4}\right)\right]$
7. $\frac{71}{4} - \frac{19}{4}i$
8. $-\frac{8}{25} - \frac{31}{25}i$
9. $\frac{1}{\sqrt{2}}, \frac{3\pi}{4}$
10.
11. x = 3 , y = -1
12. x = 1 , y = -4 or x = -1 , y = -4
13. (i) $\frac{3}{2}i$, $\frac{4}{3}i$ (ii) $-3i, \frac{2}{3}i$

$(iii) 4-3i, 3+2i, (iv) \sqrt{2i}, -2i (v) \frac{3+i}{2}, 3i$

14. $(i)\pm(1-3i),(ii)\pm(2+3i),(iii)\pm\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)$

ASSIGNMENT No. 6 (PERMUTATIONS AND COMBINATIONS)

1) 23 2) 35 3) 9 4) 3 5) 60 6) 86400 7) 60 8)(i) 86400 (ii) 28800

9) (i) 720 (ii) 24 (iii) 240 (iv) 480 10) (i) 30 (ii) 40 (iii) 46
11) 60480 12) 360 13) 15 15) 18

ASSIGNMENT No. 7 (BINOMIAL THEOREM)

1) (i) $(-1)^r\frac{11!}{r!\times(11-r)!}x^{3r}$ (ii) $(-1)^r\frac{12!}{r!\times(12-r)!}x^{24-3r}$ 2) $\frac{8064}{5}x$ 3) $-\frac{5040}{x}$ 4) $\frac{840}{x^2}$

5) 18564 7) m = 1 8) (i) 1760 (ii) 1215 9) (i) -8064 (ii) -540 11) n = 11 , r = 7
15) -6

ASSIGNMENT NO. 8 (SEQUENCES AND SERIES)

1) 550 2) $y^2=xz$ 3) 4 , 8 , 16 or -4, 8, -16 4) 10th term 5) 1024 6) $2(2^{20}-1)+\frac{5^{20}-1}{4}$

7) $\frac{7}{3},\frac{8}{3}$ 9) 31 : 51 10) $n=6$ 11) 2, 6,18 or 18, 6, 2

12) 3, 5, 7 or 15, 5, -5 13. i) $\frac{n(2n^2+3n+7)}{6}$ (ii)$3(2^n-1)-n$

14) (i) $\frac{n(n+1)(n+2)(3n+5)}{48}$ (ii) $\frac{n(n+1)(n^2+3n+1)-2n}{2}$

15. $\frac{2}{3}$

16. $5,\frac{10}{3},\frac{20}{9},\frac{40}{27},....$

17. $10,8,\frac{32}{5},....$

ASSIGNMENT NO. 9 (STRAIGHT LINES)

1) $x\cos\frac{4\pi}{3}+y\sin\frac{4\pi}{3}=6$, distance = 6 2) $\frac{17}{2\sqrt{13}}$ units 3) no 4) $x=10$

5) $x+y=\pm5\sqrt{2}$ 6) $x=-4$ 7) (-7, 4) 8) $3x+7y=84$ 9) $3x+4y+17=0$
10) (i) $13x-y-9=0$ (ii) $3x-y+1=0$ (iii) $3x-y-4=0$
11) $45^\circ,45^\circ,90^\circ$ 12) $12x+5y=60$, $5x+12y=60$ 14) $(1,-3)$
15) 4x – 3y + 16 = 0, 4x – 3y – 14 = 0
16) $x+3y-19=0$, $3x-y-7=0$ 17) $\left(\frac{31}{10},\frac{-3}{10}\right)$ 18) (-1, -14)
19) x + y -5 = 0 20) $4x+5y-1=0$ (21) $2x-3y+14=0$, (22) $(1,-5)$

ASSIGNMENT NO. 10 (CONIC SECTIONS)

1) $5x^2+y^2=5$ 2) $x^2+y^2+6x-4y-3=0$

3) 7 units 4) $4x^2+4y^2-12x+16y-21=0$ 5) $x^2+y^2-10x+6y+9=0$, Centre $(5,-3)$ and radius $r=5$

6) $x^2+y^2-17x-19y+50=0$

7) (i) F $\left(-\frac{3}{2},0\right)$; axis y = 0; directrix x = $\frac{3}{2}$; LR = 6

(ii) F$(0,2)$; axis x = 0; directrix y = -2 ; LR = 8 8) (i) $x^2=-16y$ (ii) $3y^2=4x$

9) (i) F$(\pm 3,0)$, V$(\pm 5,0)$, length of major axis = 10, length of minor axis = 8 , $e=\frac{3}{5}$ and length of LR = $\frac{32}{5}$. (ii) F$(0,\pm\sqrt{7})$, V$(0,\pm 4)$, length of major axis = 8, length of minor axis = 6 , $e=\frac{\sqrt{7}}{4}$ and length of LR = $\frac{9}{2}$. 10) (i) $3x^2+4y^2=12$ (ii) $\frac{4x^2}{81}+\frac{4y^2}{45}=1$ or $\frac{4x^2}{45}+\frac{4y^2}{81}=1$ 11) (i) F $\left(\pm\frac{\sqrt{61}}{2},0\right)$, $V(\pm 3,0)$, $e=\frac{\sqrt{61}}{6}$, $LR=\frac{25}{6}$

(ii) F$\left(0,\pm\frac{\sqrt{5}}{4}\right)$,V$\left(0,\pm\frac{1}{4}\right)$, $e=\sqrt{5}$, LR = 2 12) (i) $\frac{x^2}{32}-\frac{y^2}{224}=1$ *or* $-\frac{x^2}{224}+\frac{y^2}{32}=1$

(ii) $16x^2-2y^2=1$

ASSIGNMENT NO. 11 (THREE DIMENSIONAL GEOMETRY)

1)(b) 2) $(0,1,0)$ *and* $(0,5,0)$ 3) $(-3,-14,19)$5) 3:2 externally 6) 1:3 externally $(0,-1,3)$

7) $(2,5,2)$ 9) $(6,-1,8)$ 10) $(0,0,5)$

ASSIGNMENT NO. 12 (LIMITS AND DERIVATIVES)

1) (i) $\frac{-1}{4}$ (ii) 3 (iii) 1 (iv) 1 (v) $\frac{1}{12}$ (vi) $\frac{-1}{4}$ 2) 7 3) (i) $3x^2-2ax-b$

(ii) $x^3\sec^2 x+3x^2\tan x$ (iii) $\frac{-x^2-6x+1}{(x^2+1)^2}$ 4) 9 5) (i) $\frac{15}{11}$ (ii) 2 (iii) 4(iv) $\frac{4}{9}$ (v) -4

(vi) $\frac{1}{16\sqrt{2}}$ 6) (i) $1+\frac{1}{x^2}$ (ii) $2x-2x^{-3}$ (iii) $\sec(x+1)\tan(x+1)$(iv) $\frac{x\cos x-\sin x}{x^2}$ (v) $-x\sin x+\cos x$ 7) (i) $\frac{2\sin x}{(1+\cos x)^2}$ (ii) $\frac{x+\sin x}{1+\cos x}$ (iii) $3x^2+\frac{1}{x^2}-1-\frac{3}{x^4}$ (iv) $-\frac{2}{(x+1)^2}-\frac{3x^2-2x}{(3x-1)^2}$ 8) (i) limit does not exist (ii) $\frac{1}{2}$ (iii) 1 9) $a=5$ *and* $b=-39$ 10) (i) 1 (ii) 0 (iii) 5 (iv) (log 2)(log 3)

ASSIGNMENT NO. 13 (PROBABILITY)

2) 25/28 3) 63/190 4) 0.08 5) 13/18 6) 40% or 0.4 7) i)4/5 or 0.8 ii) 0.5 8) 316/435
9) i) 5/9 ii) 4/9 iii) 2/3 10) 286/20825 11)1/5040 12) 2/3